ELASTIC BEHAVIOR OF METAL INTERFACES

Arthur J. Coyle
and
Herman A. Stromberg

ELASTIC BEHAVIOR OF METAL INTERFACES

by

Arthur J. Coyle, Lieutenant, U. S. Navy
Herman A. Stromberg, Jr., Lieutenant (j.g.), U. S. Navy
B. S., U. S. Naval Academy, 1949

Submitted in Partial Fulfillment
of the Requirements for the
Degree of Naval Engineer
from the
Massachusetts Institute of Technology
1954

8854
on spine:
COYLE
1954
THESIS
C77

Letter on front cover:

ELASTIC BEHAVIOR OF METAL
INTERFACES

Arthur J. Coyle
and
Herman A. Stromberg

Thesis
C77

[illegible] BEHAVIOR OF METAL INTERFACES

by

Arthur [illegible]. Coyle, Lieutenant, U. S. Navy
[illegible]erg, Jr., Lieutenant (j.g.), U. S. Navy
B. S., U. S. Naval Academy, 1949

Submitted in Partial Fulfillment
of the Requirements for the
Degree of Naval Engineer
from the
Massachusetts Institute of Technology
1954

ELASTIC BEHAVIOR OF METAL INTERFACES

by
Arthur J. Coyle, Lieutenant, U. S. Navy
Herman A. Stromberg, Jr., Lieutenant (j.g.), U. S. Navy

Submitted to the Department of
Naval Architecture and Marine Engineering
May 24, 1954
in partial fulfillment of the requirements for the
degree of Naval Engineer

ABSTRACT

The object of this thesis is to investigate the reported existence of abnormal elastic effects in metal interfaces.

In general two separate methods of investigation were conceived and used. The first employed cylindrical specimens with truncated conical ends placed between flat surfaces. The second, devised to eliminate any possible elasticity contributed by the flat surfaces beyond the area of contact, employed two identical annular specimens placed end on end.

There was good correlation between the two methods since both produced excessive elastic angles of twist. Further, the following unusual results were observed. As the normal stress was increased in the interface, the amount of excess twist for a given torque increased with the degree of surface finish until a transition range was reached after which excess twist increased with the roughness of the surface.

It is concluded that these effects are caused by the asperities acting as cantilevers. This offers an explanation for the effect of normal stress and surface finish on the elastic angles of twist that is in agreement with the experimental data.

In view of the above, it is recommended that further studies be made using metals of different physical characteristics.

Thesis Supervisor: Brandon G. Rightmire
Title: Associate Professor of
Mechanical Engineering

25071

Massachusetts Institute of Technology
Cambridge 39, Massachusetts
May 24, 1954

Professor L. F. Hamilton
Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge 39, Massachusetts

Dear Sir:

In accordance with the regulations of the faculty, we submit herewith a thesis entitled Elastic Behavior of Metal Interfaces in partial fulfillment of the requirements for the degree of Naval Engineer.

Respectfully,

Arthur J. Coyle
Lieutenant, U. S. Navy

Herman A. Stromberg, Jr.
Lieutenant (j.g.), U. S. Navy

ACKNOWLEDGMENTS

The authors wish to express their gratitude to the personnel of the Lubrication Laboratory for their interest and help. Particularly to Professor B. G. Rightmire for his encouragement, advice, and boundless patience; and to Professor I. M. Feng, Doctor E. Rabinowicz, Mr. James Swartwout, and Mr. Jacques Bonneville, for their kind assistance.

TABLE OF CONTENTS

I. INTRODUCTION 1

II. PROCEDURE 3

III. RESULTS 7

IV. DISCUSSION OF RESULTS. 27

V. CONCLUSIONS AND RECOMMENDATIONS 32

VI. APPENDIX 33

  A. Details of Procedure 34

  B. Summary of Data and Calculations 41

  C. Supplementary Discussion 60

  D. Original Data 67

  E. Definition of Symbols 78

  F. Bibliography 80

INTRODUCTION

Although much information is available concerning the elastic behavior of bulk metals, data on the elastic behavior of metal interfaces is outstanding by its absence.

In a paper (1), Tomlinson, Thorpe, and Gough reported observing deflection between two metal rings when subjected to a known torque far in excess of that predicted by theory and the excess was greater than could be accounted for by experimental error. Since this phenomenon was not directly connected with the objective of their work it was not investigated further. An element of uncertainty was associated with this report since the deflection was measured by means of the same arm with which the torque was applied. It was possible that the excessive deflection reported could have been caused by bending of the torque arm. Nonetheless, the report of this phenomenon was sufficiently enticing to invite further investigation.

Bowden and Tabor (2) show that the actual area of contact between two surfaces is different from the apparent. This difference is caused by the asperities to be found on all surfaces.

Herring and Galt (3) have shown that very small metal specimens (i.e., metal whiskers) can tolerate much larger strains without slip than the bulk metal.

Inasmuch as the volume of an asperity is small, it is considered possible that, like the metal whiskers, its elastic limit is high. A study of the literature did not reveal any investigation of this

possibility other than the report by Tomlinson, Thorpe, and Gough mentioned in the second paragraph. To us this seemed to present a challenge that was interesting, provocative, and stimulating.

PROCEDURE

Since two experimental arrangements were used in this work, it would be best to present these schemes separately.

The first scheme employed is illustrated by Figure XIX in which the solid steel specimens tested were machined from SAE 1020 cold rolled steel stock. The ends of most of the specimens were beveled to form truncated cones which reduced the contact surfaces and thereby increased the normal stress in the interfaces. The contact surfaces were lapped to the desired finish with emery polishing paper of varying degrees of roughness. The specimens were firmly held in a vee-type block during the lapping process to make the test surfaces perpendicular to the specimen axis. The quality of the surface finish was checked microscopically using a magnification of 165x.

When the desired finish was obtained, the average diameter, d, and bevel height, ℓ, (Figure XX) were measured by a microscope equipped with an optic vernier permitting accuracy to within .00007 of an inch. The ends of the specimens were made identical with respect to dimensions and surface finish.

The next step was to locate the torque lever exactly at the midpoint of the specimen to insure that one half the applied torque would act on each interface. This was accomplished by measuring the overall length and using a depth gage in positioning the lever which permitted accuracy to the nearest .001 inch.

The indicator arm was then fastened on the specimen with four

set screws so that the plane of the set screws and hence, the indicator, was perpendicular to the specimen axis. The set screws used were pointed to obtain a knife edge line from which the twist was transmitted. The screws were also barely tightened by hand to reduce the indentations to mere pin points.

With the positioning collar resting on the torque lever, the contact surfaces of the specimen and hardened plates were cleaned with solvent. The specimen was then placed on the lower plate in the Riehle testing machine. The upper plate, which was bolted to the movable head of the testing machine, was then lowered to a point slightly above the specimen. The vertical was then checked by inserting a feeler gage between the positioning collar and the upper plate. An axial load of one hundred pounds was then applied to anchor the specimen during the final adjustments to the apparatus.

Finally the frictionless pulleys were located so that the silk threads, which transmit force to the torque lever, were perpendicular to the lever in the vertical and horizontal planes.

For each run on a specimen a constant normal load was applied by the testing machine. Torque was next applied to the specimen by adding equal weights to the pans and the resulting twist read. After each reading of twist, the torque was removed to see if the indicator returned to the zero position. Increasing torques were applied until the indicator failed to return to zero which showed that slip or permanent deformation occurred in the interface ending the run. The normal load was increased before each of the succeeding

runs on the same specimen until sufficient data was obtained.

Twist was read by measuring the motion of the indicator arm under a microscope equipped with an optic vernier. The indicator was double-ended to avoid any possibility of an unbalancing moment. The first indicator arm used was seven inches long and the magnification of the microscope was such that one scale unit of the optic vernier represented .001 inch of indicator movement.

After completion of all runs on a specimen, the apparatus was dismantled and the dimension ℓ' was measured with the microscope which permitted accuracy to within .00007 of an inch

A number of specimens were tested using this scheme; however, an uncertainty existed as to how much twist, if any, could be attributed to the hardened steel plate. Consequently scheme two was devised to eliminate this unknown factor.

The second scheme employed is illustrated in Figure XXI, in which the tubular steel specimens tested were machined from SAE 1020 cold rolled stock. The specimens had an outer diameter of 1/4 inch, an inner diameter of 5/32 inch, and a length of one inch. Two specimens with the same surface finish were mounted end on, and a rod with a diameter slightly less than the inner diameter of the annuli was threaded through the apparatus as shown in Figure XXI. The upper end of the rod was threaded into a circular disc which rested on top of the upper specimen, the lower end extended for some distance below the specimens and

provided a means for attaching weights to furnish the normal load.

Two 7-1/2" double-ended indicator arms were used, on on either side of the interface. As before they were attached by pointed set screws and the distance between them measured microscopically after each run. The ends of the arms were bent as shown to facilitate reading the microscope. Deflections were obtained by reading the angle between the arms when no torque was applied and then reading the angle after the torque had been applied. A simple subtraction gave the deflection. As before, a check was made after each application of a torque to see that the arms returned to their initial setting.

Torques were applied in the same manner as in the first method.

An additional advantage obtained by use of this method was that it permitted the use of a control specimen. This consisted of using a single two inch specimen while holding all other conditions the same as they were for the runs involving two one inch specimens. The effect of the interface could thus be determined by comparing the runs on the control specimen with the runs on the other specimens.

The observed deflection was compared with the computed deflection as predicted by elastic theory. In all cases the observed was greater than the computed. This excess observed deflection which we shall hereafter refer to as residual deflection was plotted against the computed maximum tangential stress in the interface.

RESULTS

The results of this investigation are shown in Figures I - XVIII-A, inclusive.

RESULTS

The results of this investigation are shown in Figures 1 - 151, inclusive.

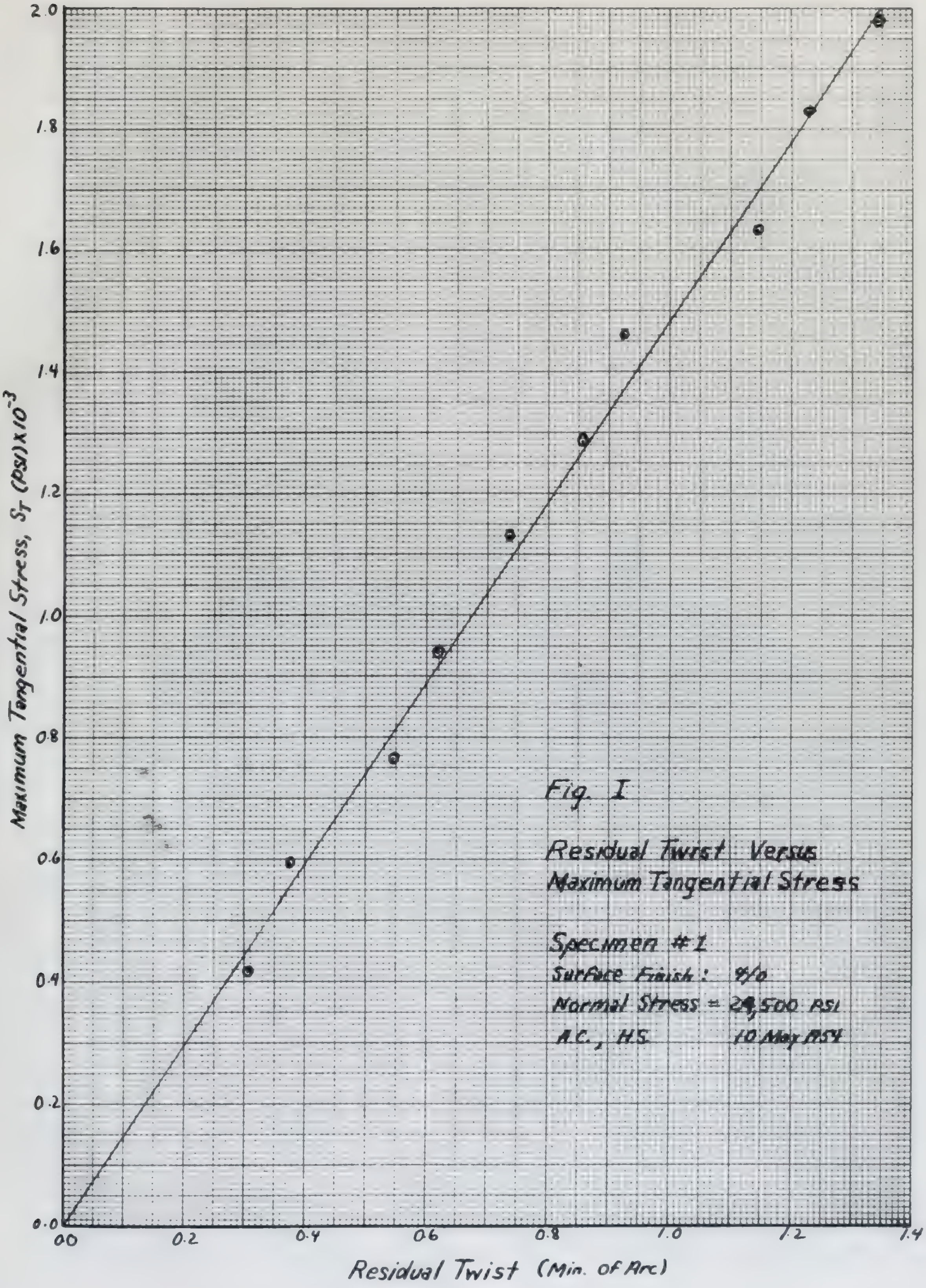


Fig. I

Residual Twist Versus Maximum Tangential Stress

Specimen #1

Surface Finish: 4/0

Normal Stress = 24,500 PSI

A.C., H.S. 10 May 1954

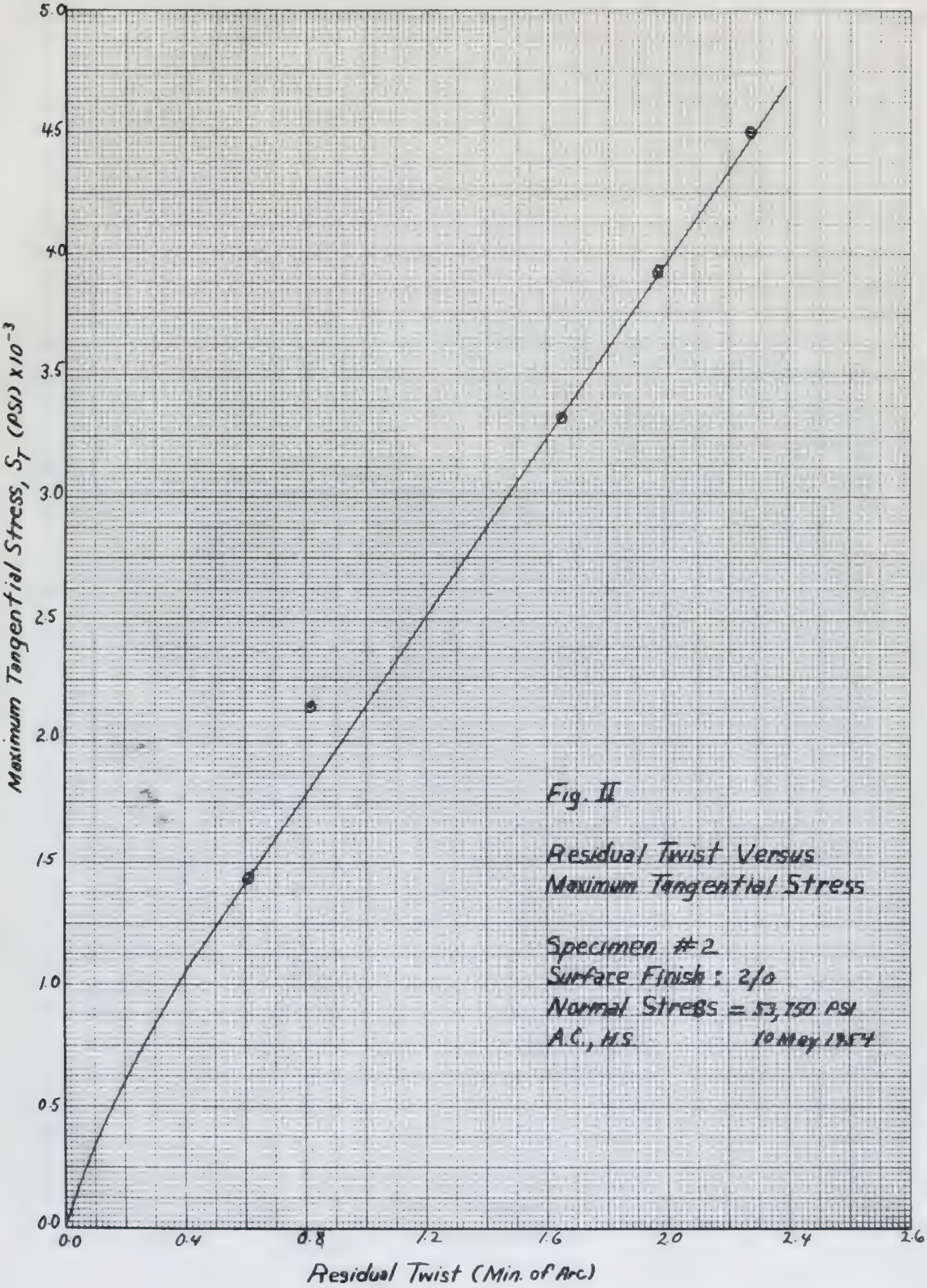


Fig. II

Residual Twist Versus
Maximum Tangential Stress

Specimen #2
Surface Finish : 2/0
Normal Stress = 53,750 PSI
A.C., H.S. 10 May 1954

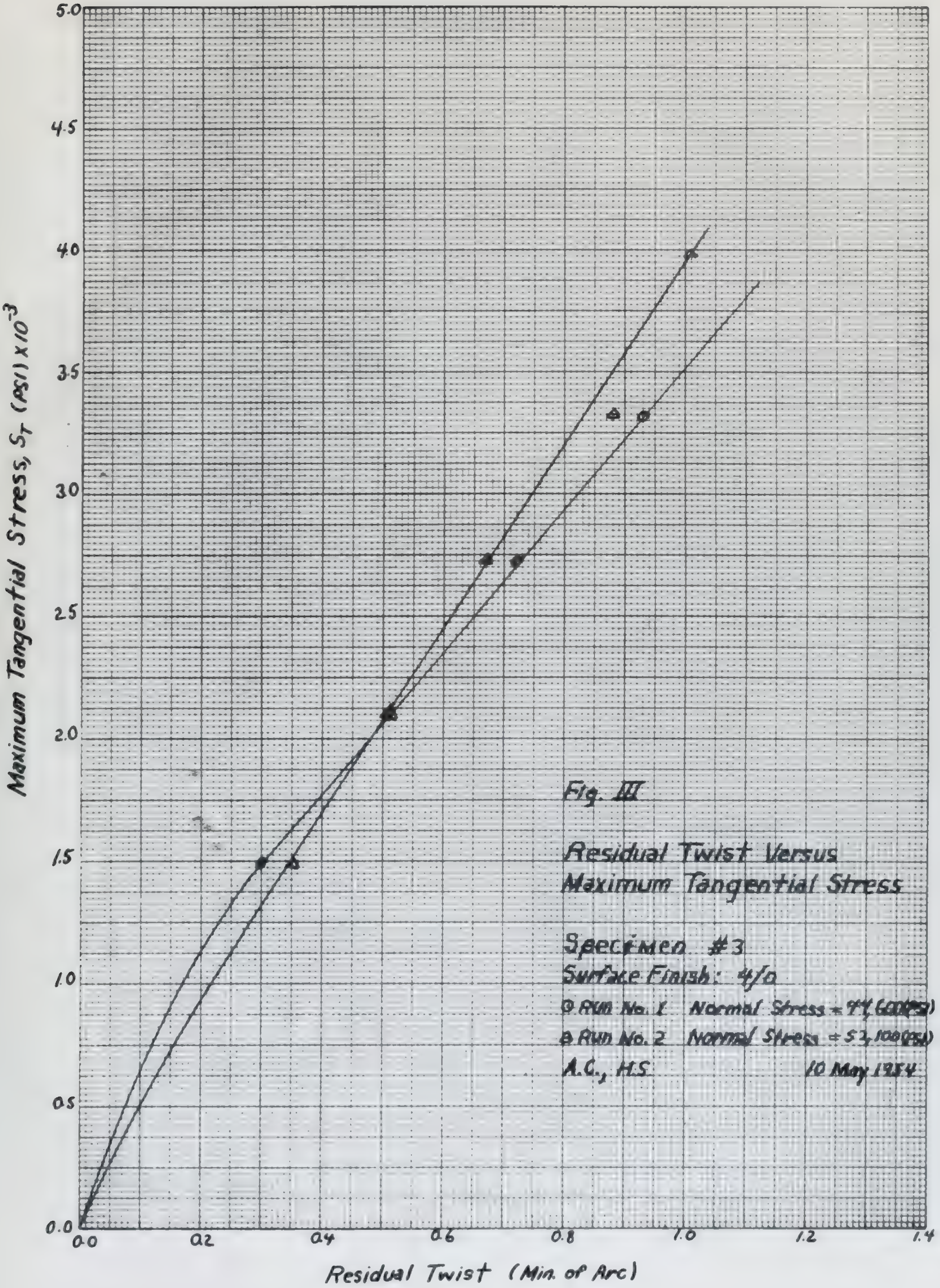


Fig. III

Residual Twist Versus
Maximum Tangential Stress

Specimen #3
Surface Finish: 4/0

A.C., H.S. 10 May 1954

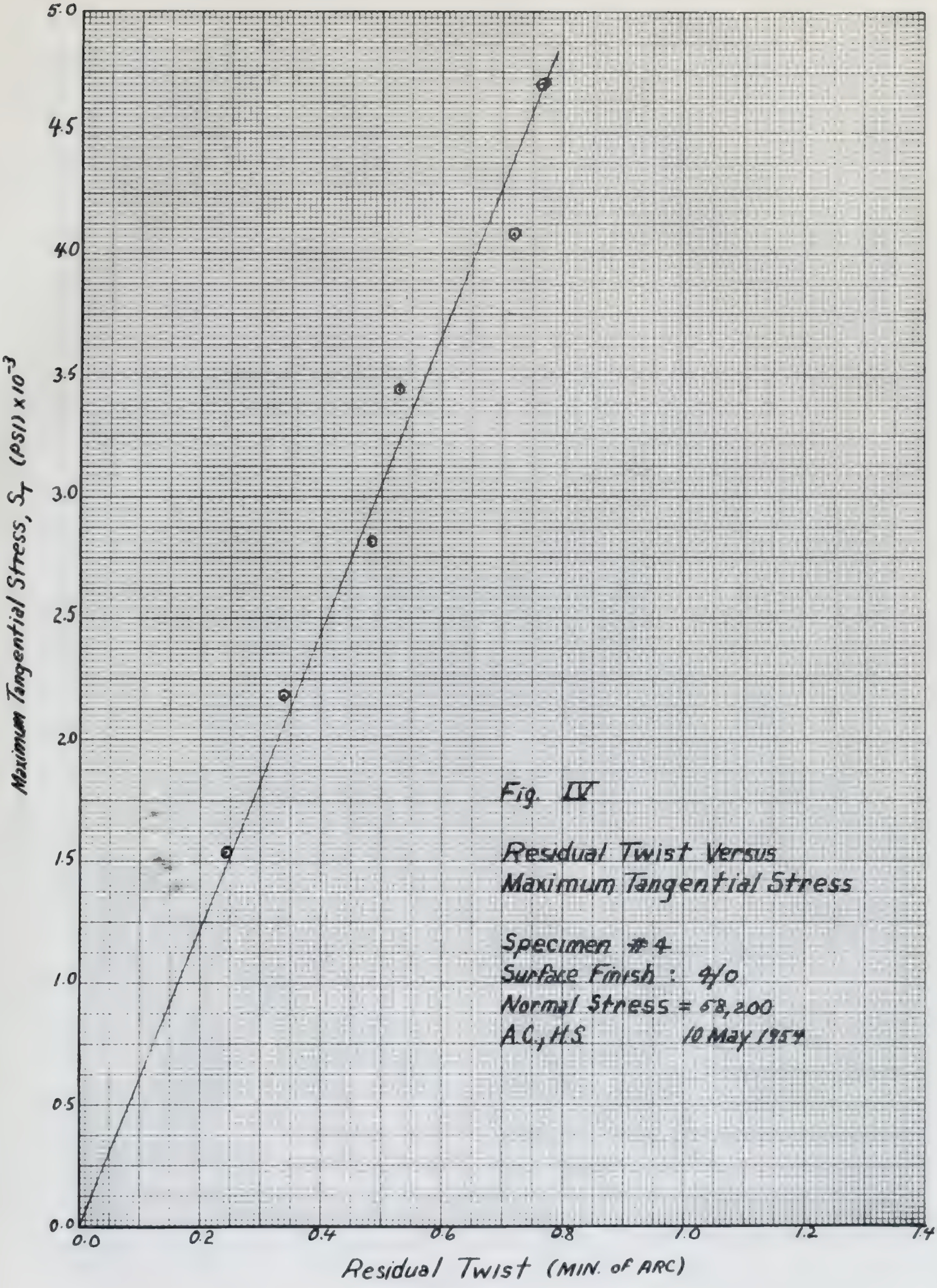


Fig. IV

Residual Twist Versus
Maximum Tangential Stress

Specimen #4
Surface Finish : 4/0
Normal Stress = 68,200
A.C., H.S. 10 May 1954

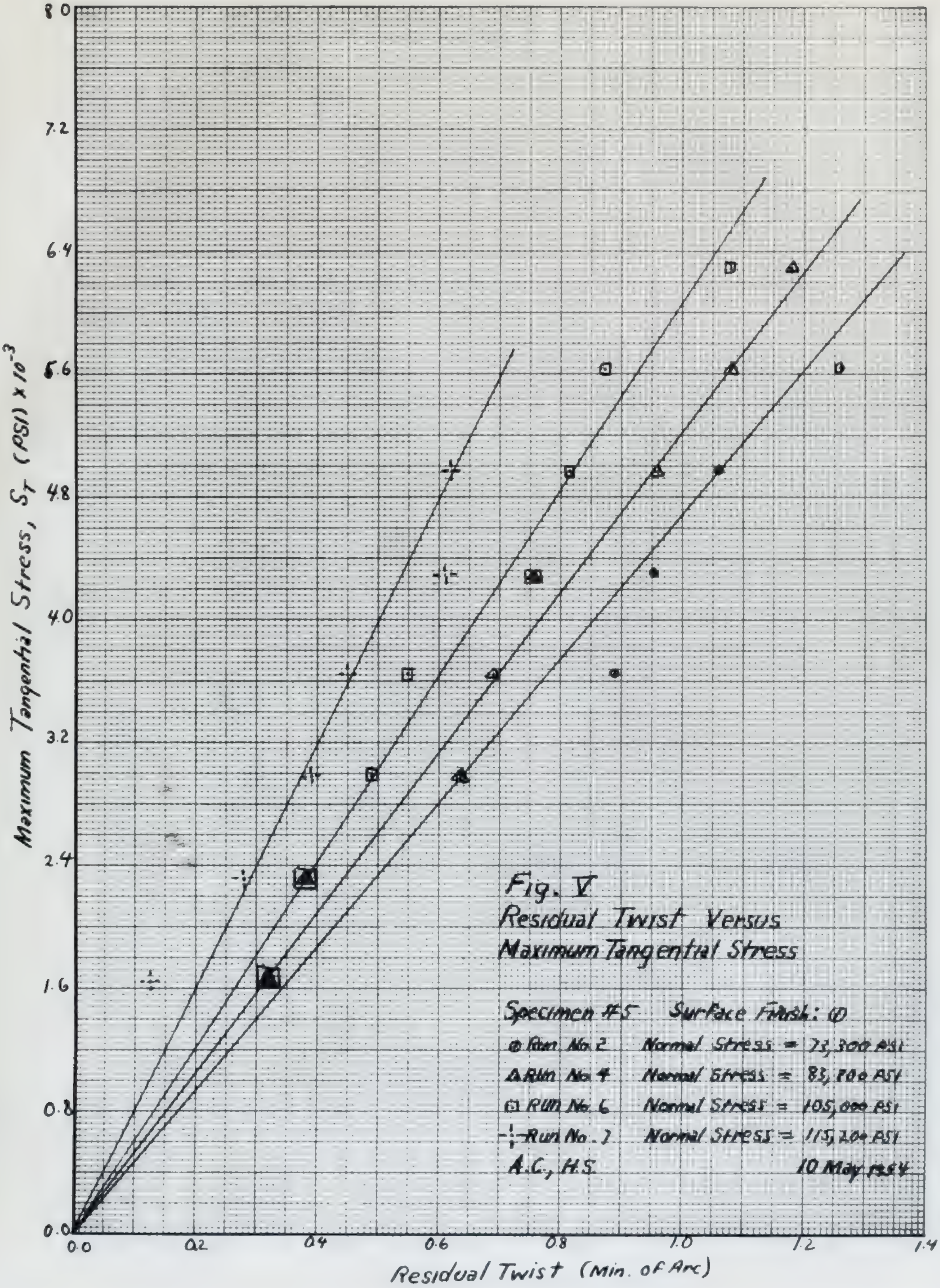


Fig. V

Residual Twist Versus Maximum Tangential Stress

Specimen #5 Surface Finish: ⦶

⦶ Run No. 2 Normal Stress = 73,300 PSI

△ Run No. 4 Normal Stress = 83,800 PSI

⊡ Run No. 6 Normal Stress = 105,000 PSI

-¦- Run No. 7 Normal Stress = 115,200 PSI

A.C., H.S. 10 May 1954

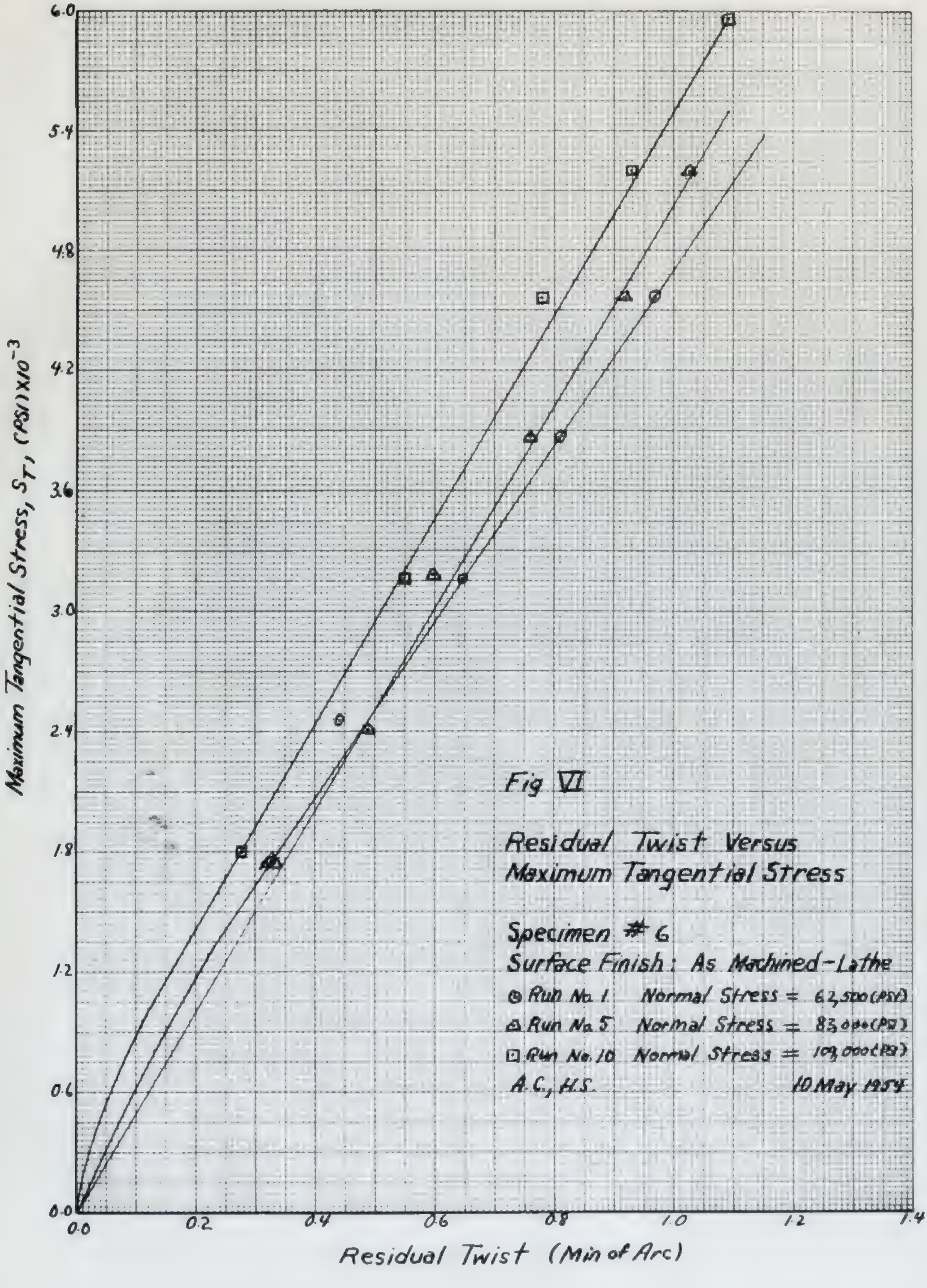


Fig VI

Residual Twist Versus
Maximum Tangential Stress

Specimen # 6
Surface Finish: As Machined-Lathe

A.C., H.S. 10 May 1954

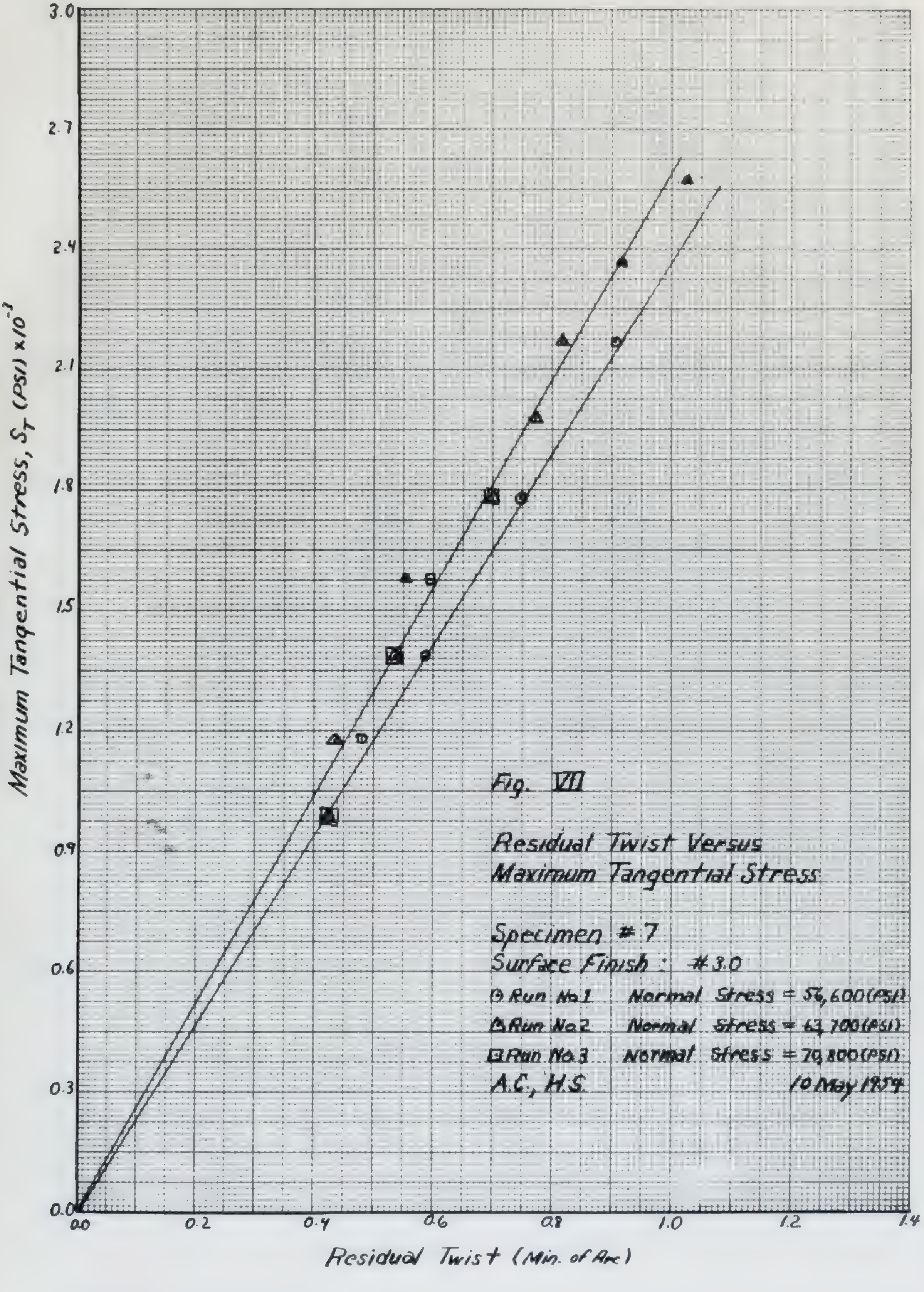


Fig. VII

Residual Twist Versus
Maximum Tangential Stress

Specimen #7
Surface Finish : #3.0

A.C., H.S. 10 May 1954

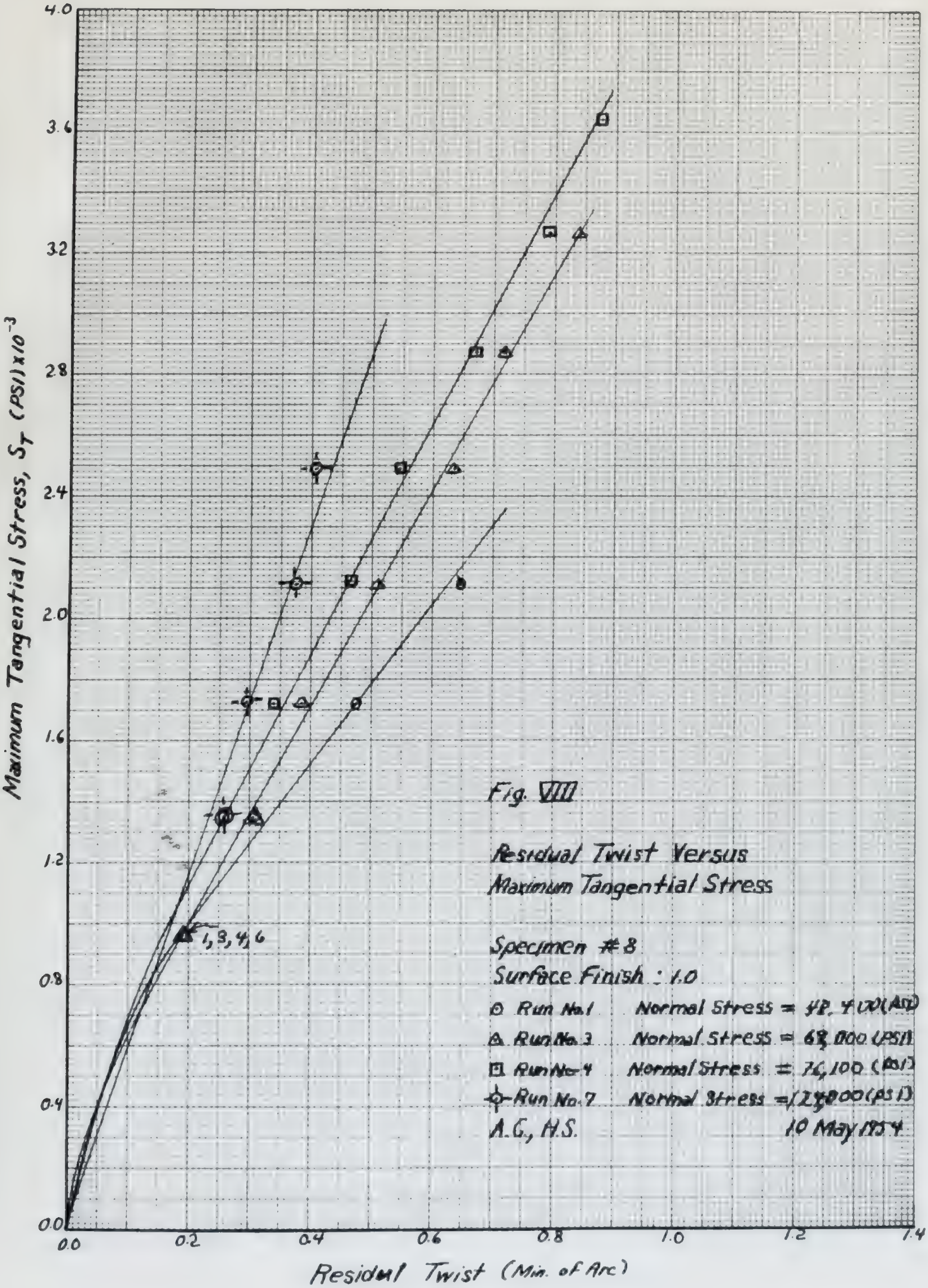


Fig. VIII

Residual Twist Versus Maximum Tangential Stress

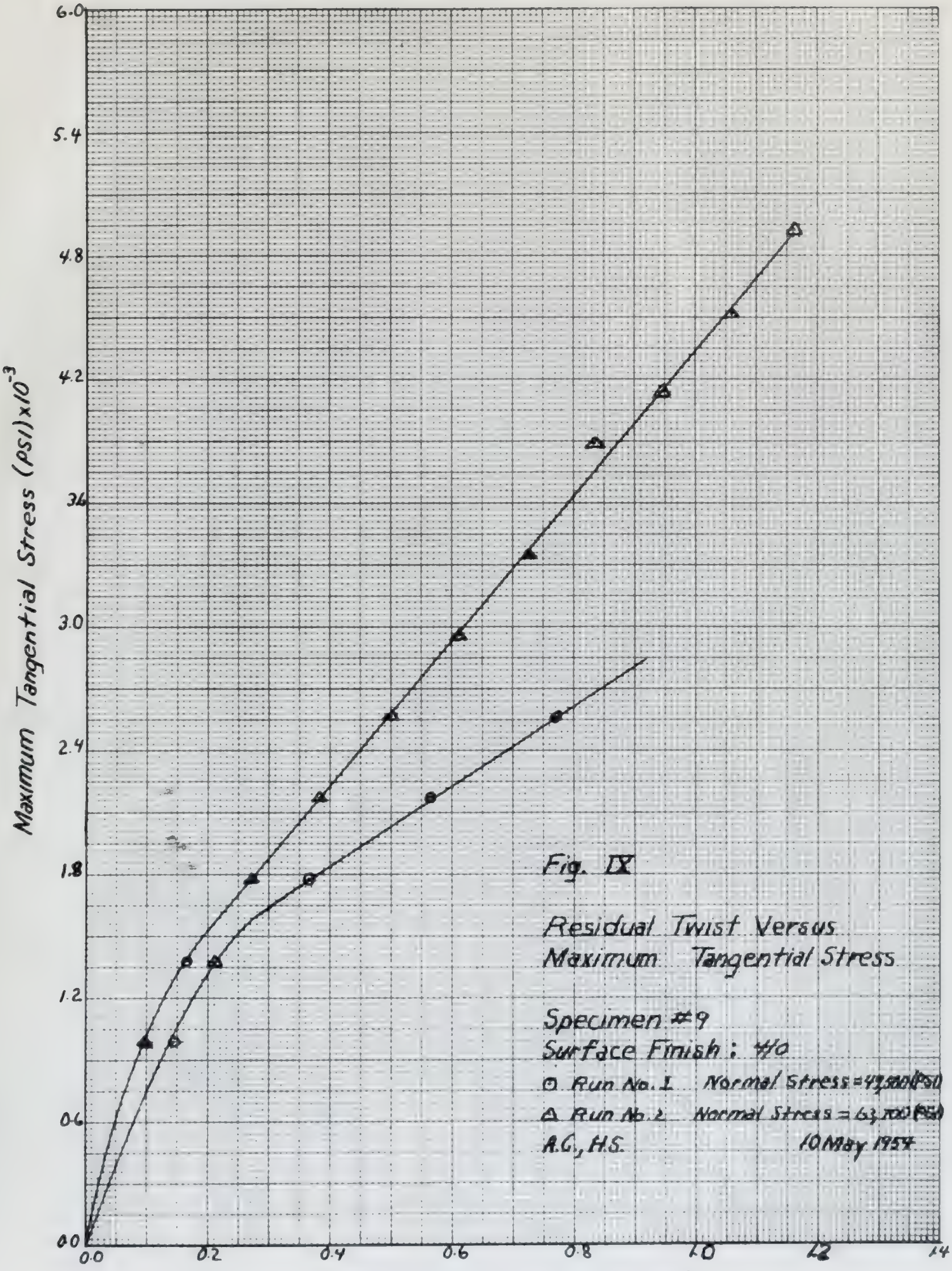


Fig. IX

Residual Twist Versus
Maximum Tangential Stress

Specimen #9
Surface Finish : 4/0
○ Run No. 1 Normal Stress = 47,500 (PSI)
△ Run No. 2 Normal Stress = 63,700 (PSI)
A.G., H.S. 10 May 1954

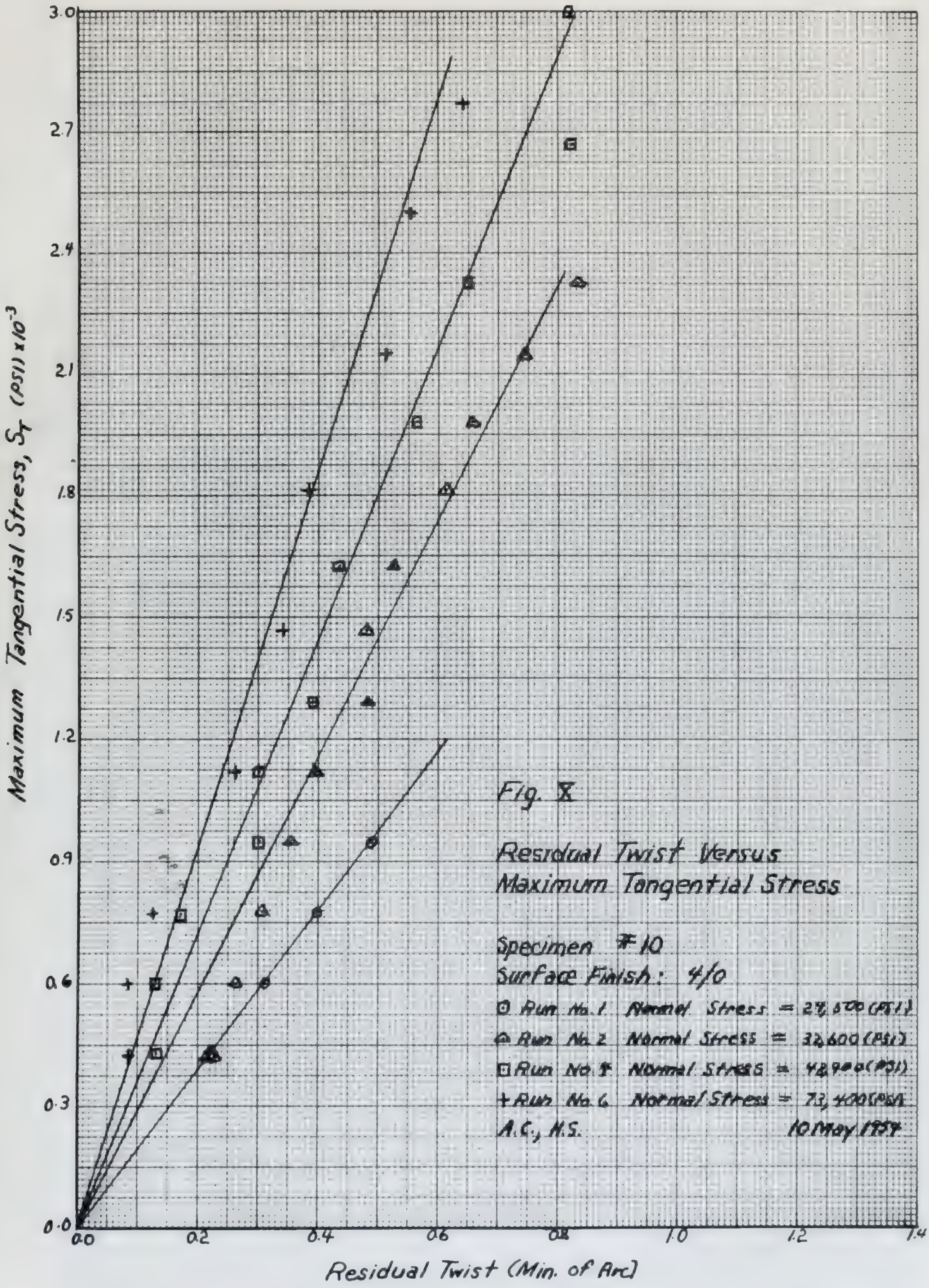


Fig. X

Residual Twist Versus Maximum Tangential Stress

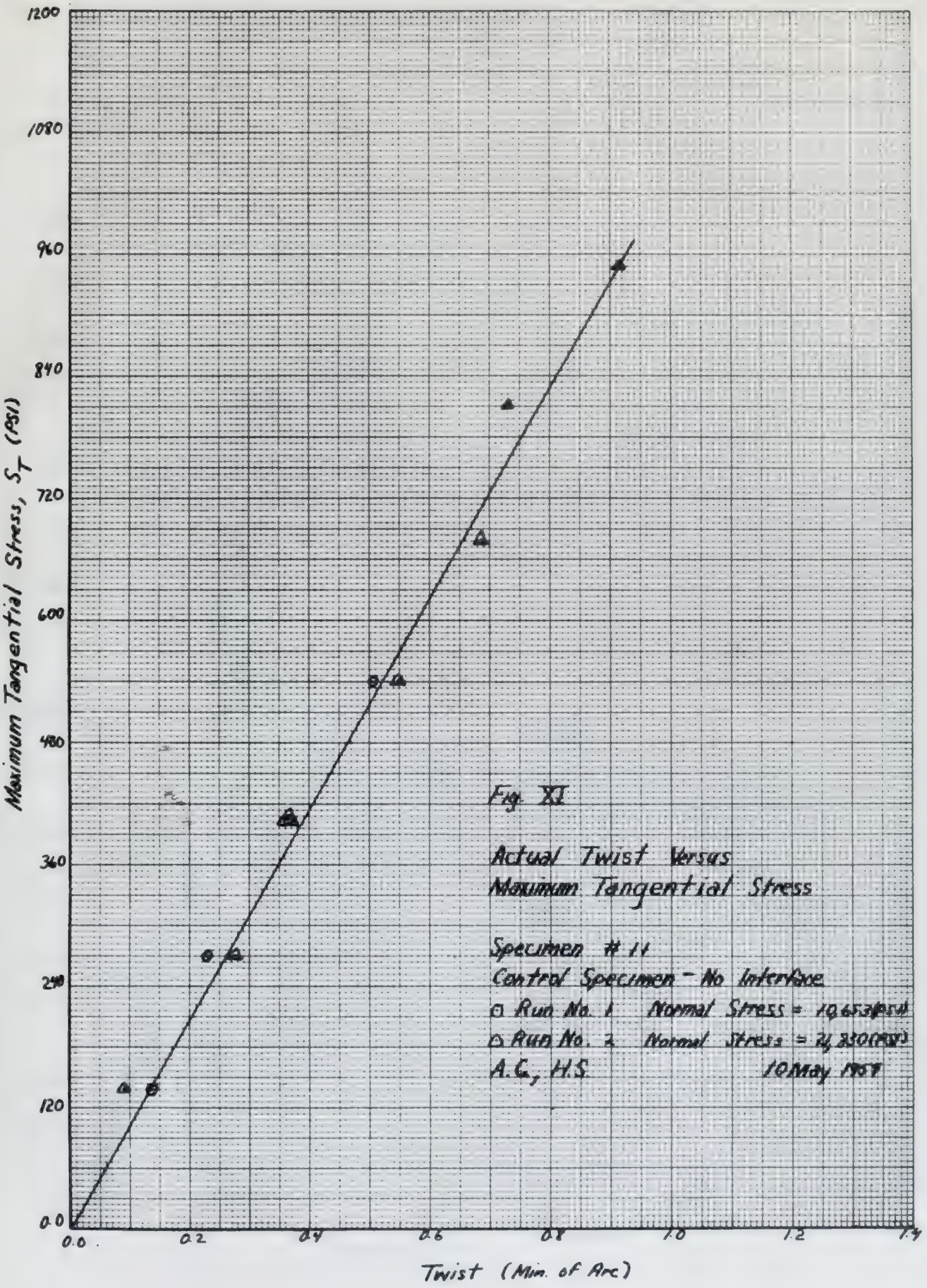


Fig. XI

Actual Twist Versus
Maximum Tangential Stress

Specimen #11
Control Specimen - No Interface
○ Run No. 1 Normal Stress = 10,653(PSI)
△ Run No. 2 Normal Stress = 21,830(PSI)
A.C., H.S. 10 May 1958

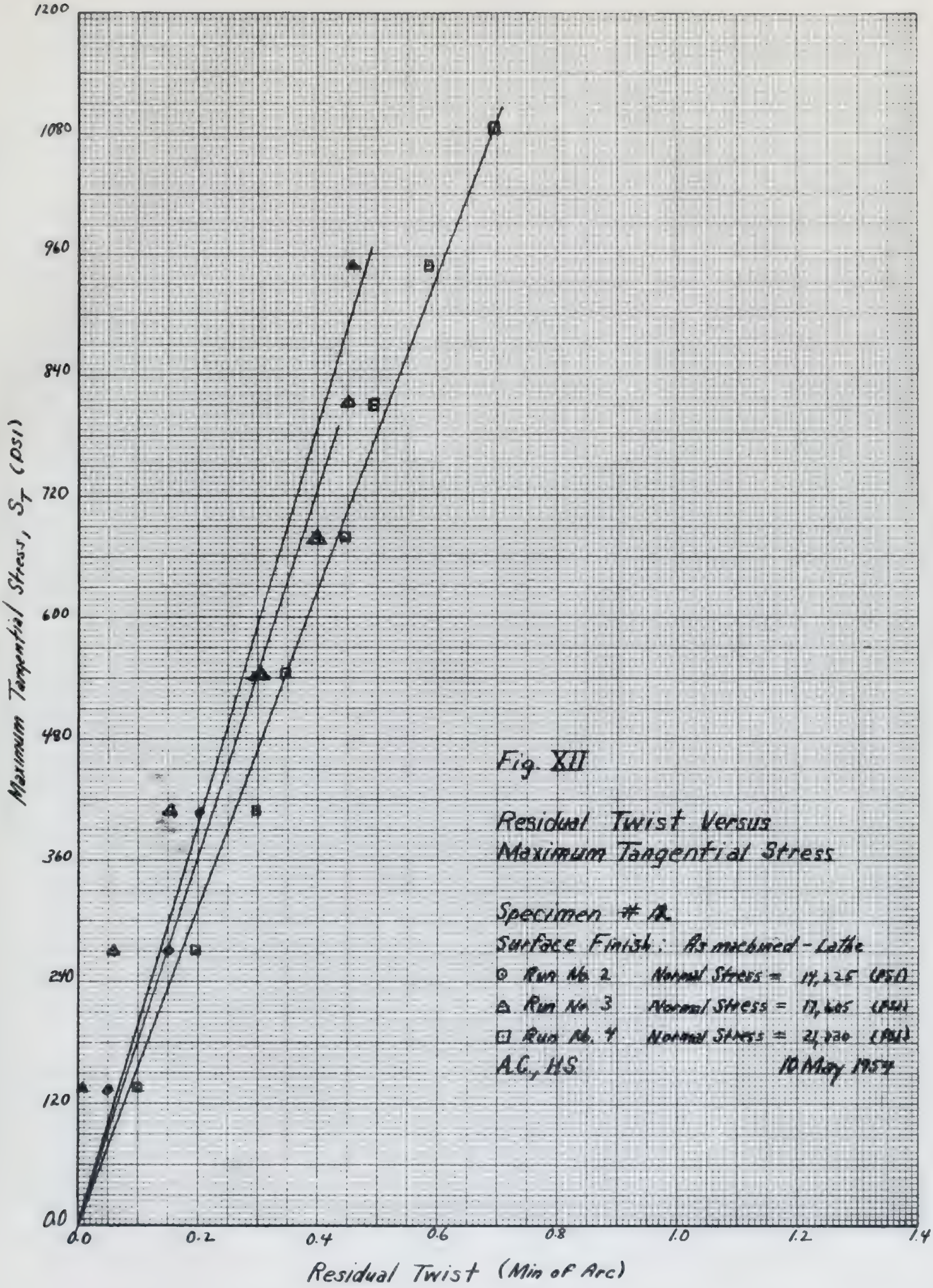


Fig. XII

Residual Twist Versus Maximum Tangential Stress

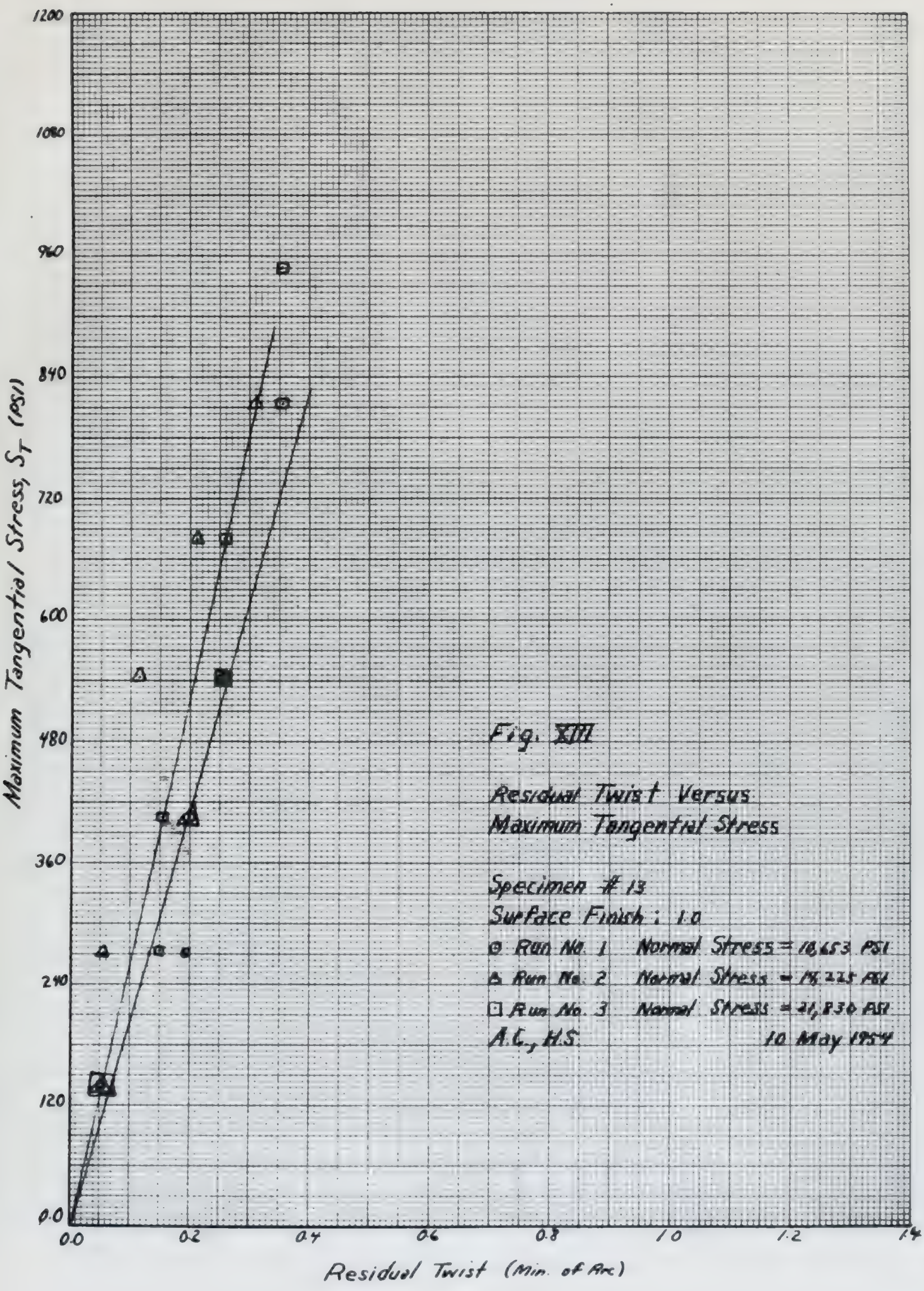


Fig. XIII

Residual Twist Versus
Maximum Tangential Stress

Specimen # 13
Surface Finish : 1.0
⊙ Run No. 1 Normal Stress = 18,653 PSI
△ Run No. 2 Normal Stress = 18,225 PSI
□ Run No. 3 Normal Stress = 21,830 PSI
A.C., H.S. 10 May 1954

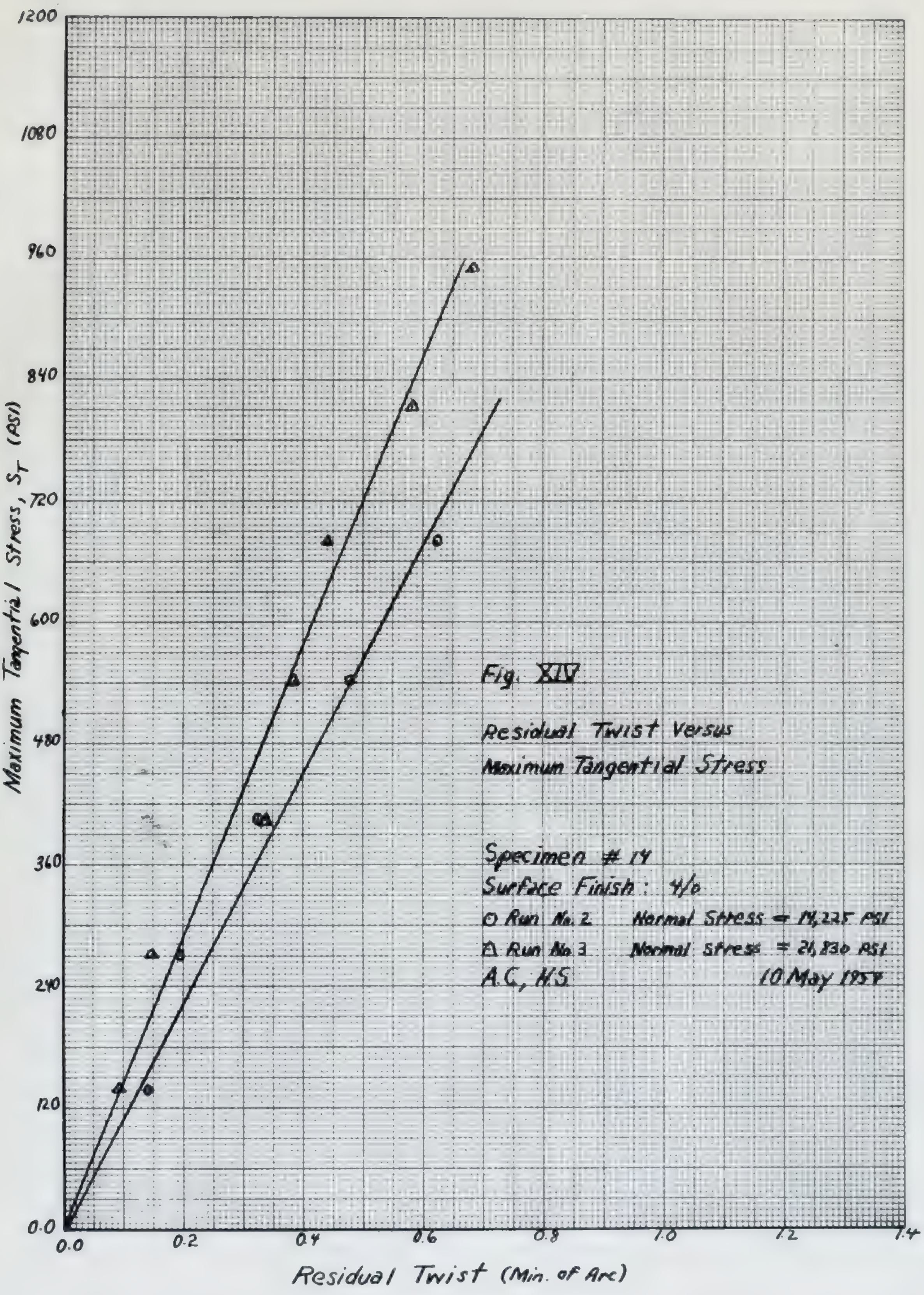


Fig. XIV

Residual Twist Versus
Maximum Tangential Stress

Specimen # 14
Surface Finish: 4/0
○ Run No. 2 Normal Stress = 14,225 PSI
△ Run No. 3 Normal Stress = 21,830 PSI
A.C., H.S. 10 May 1957

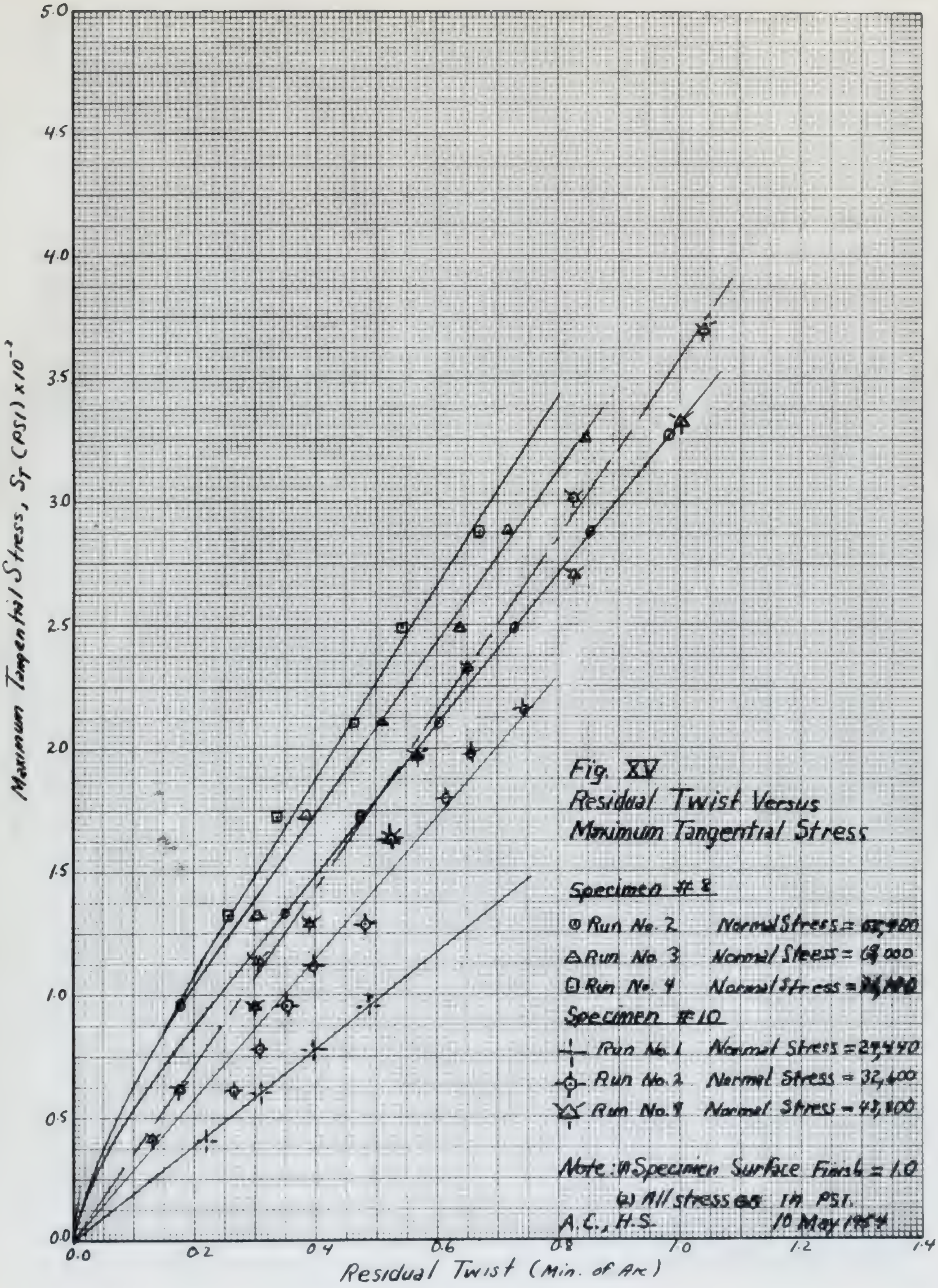


Fig. XV
Residual Twist Versus
Maximum Tangential Stress

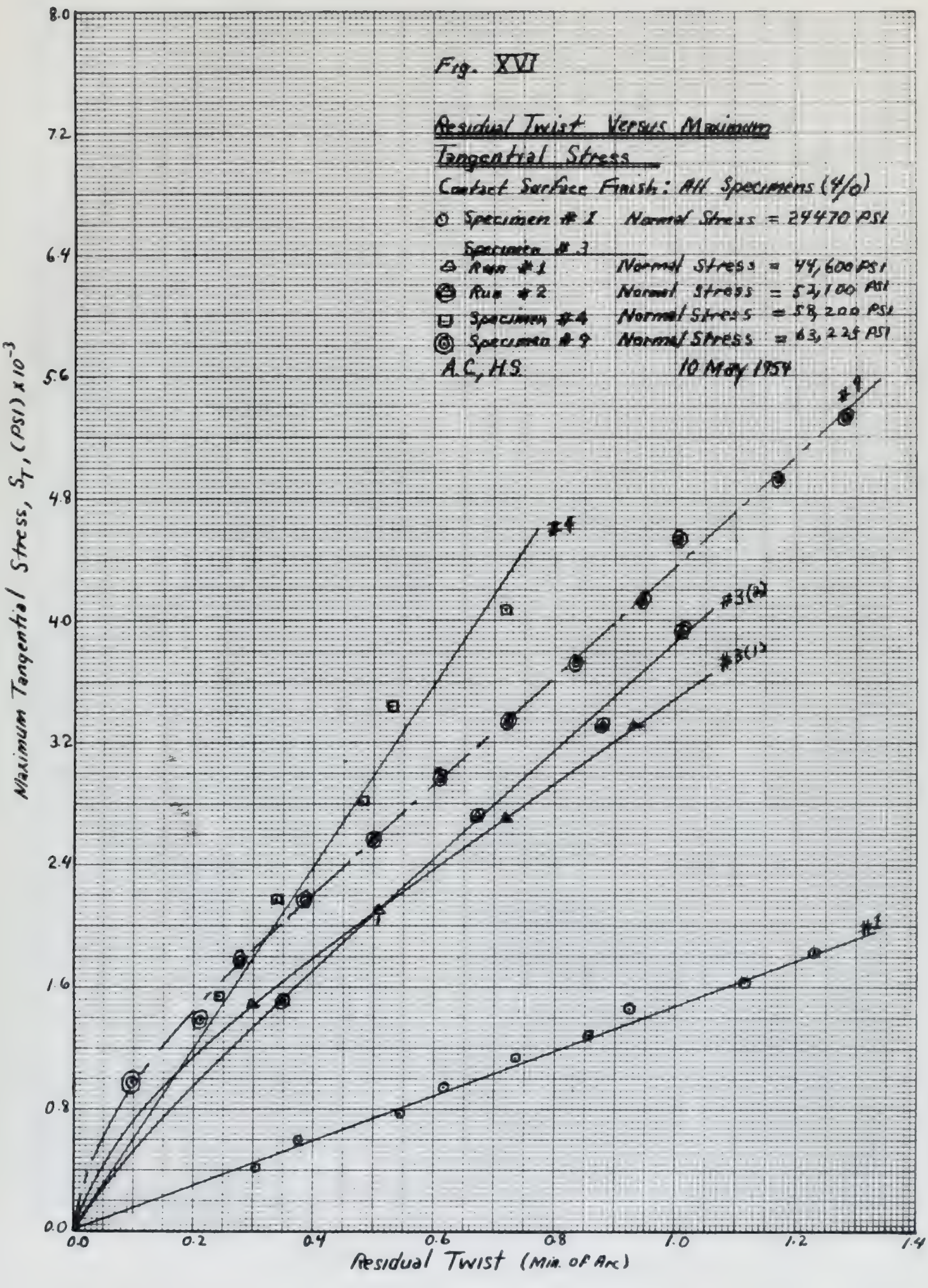


Fig. XVI

Residual Twist Versus Maximum Tangential Stress

Contact Surface Finish: All Specimens (4/0)

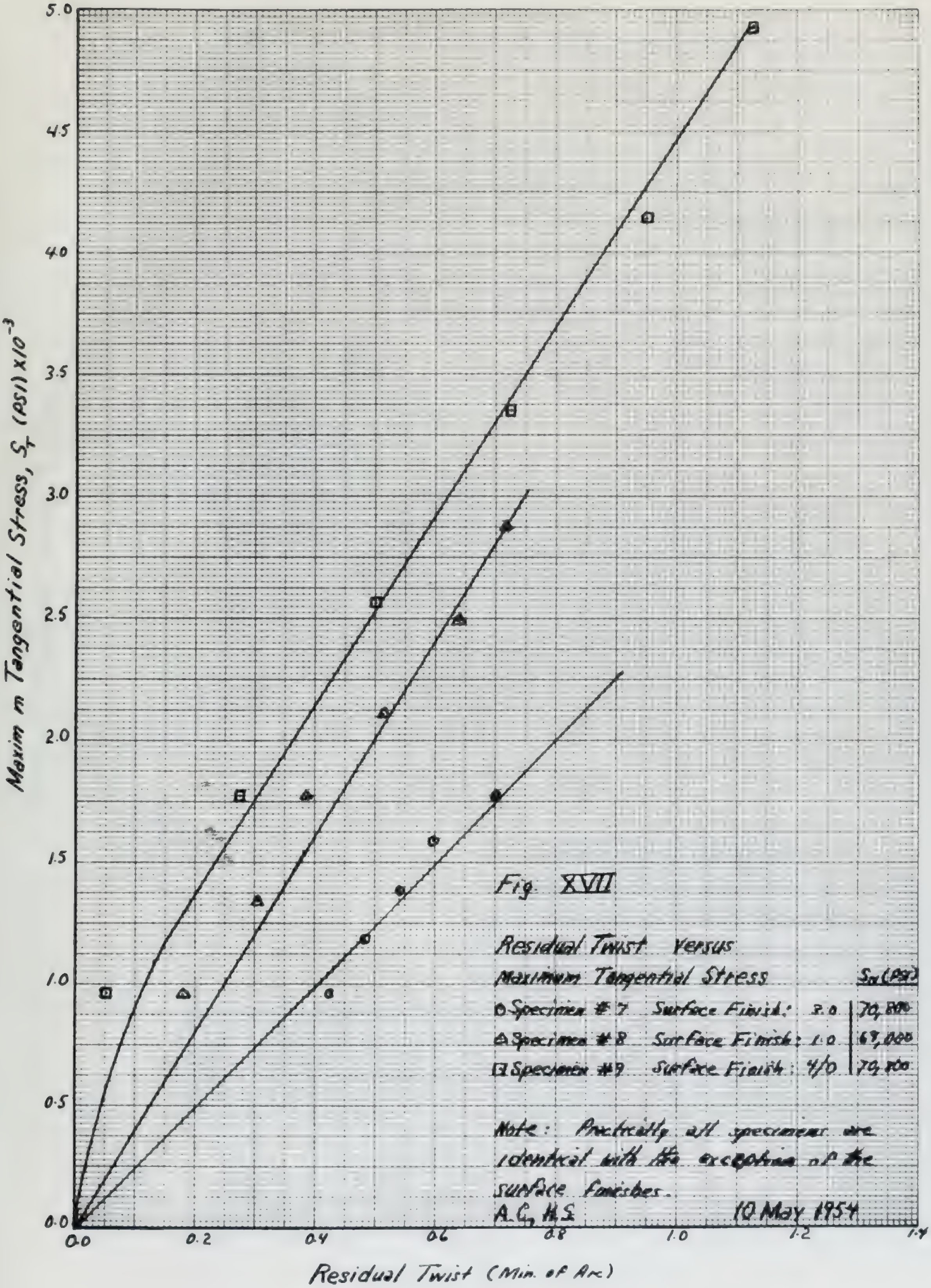


Fig. XVII

Residual Twist versus Maximum Tangential Stress

Note: Practically all specimens are identical with the exception of the surface finishes.

A.C., H.S.

10 May 1954

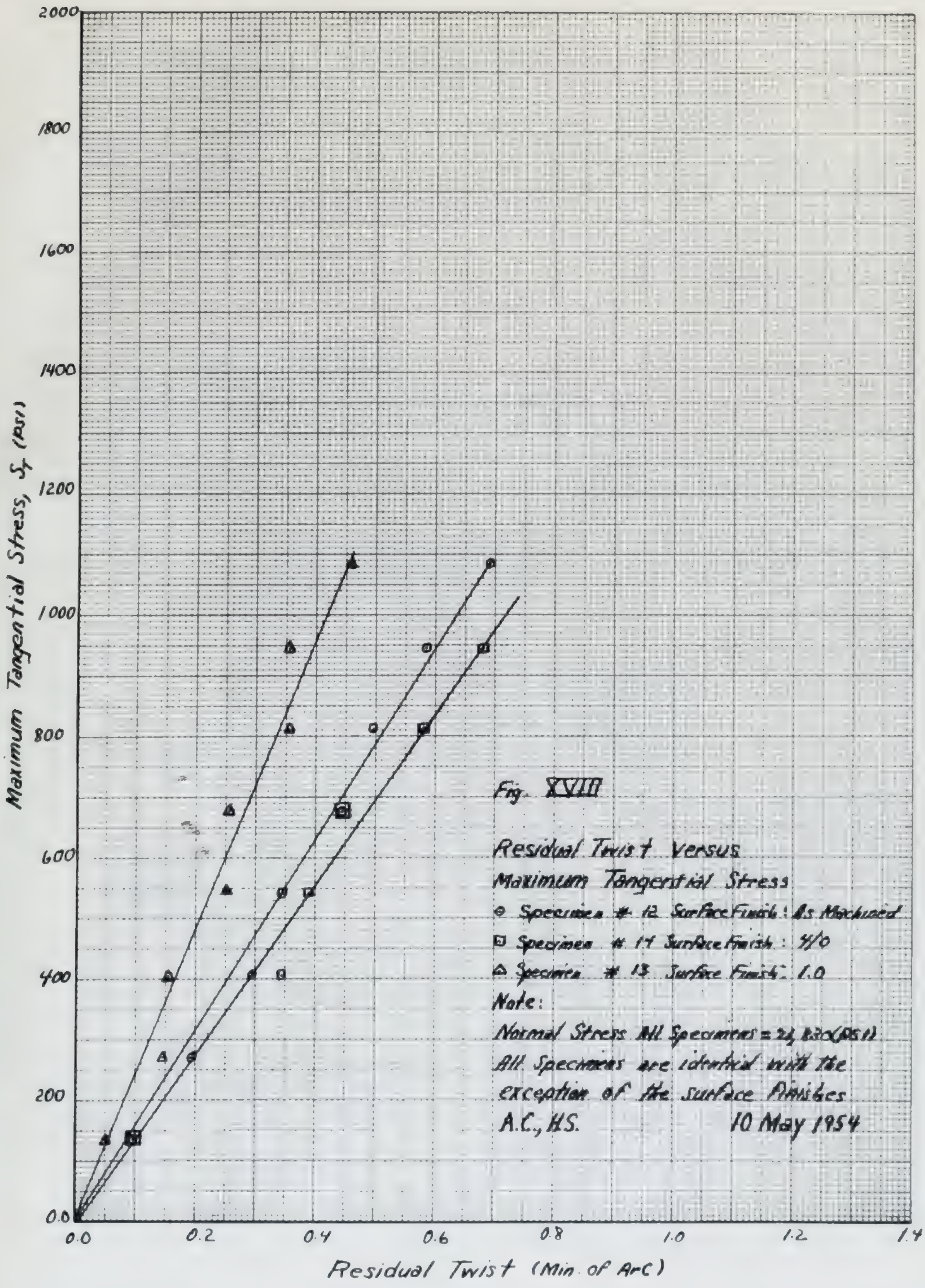


Fig. XVIII

Residual Twist Versus
Maximum Tangential Stress

⊙ Specimen # 12 Surface Finish: As Machined
⊡ Specimen # 14 Surface Finish: 4/0
△ Specimen # 13 Surface Finish: 1.0

Note:

Normal Stress All Specimens = 24,830 (PSI)
All Specimens are identical with the exception of the surface finishes

A.C., H.S. 10 May 1954

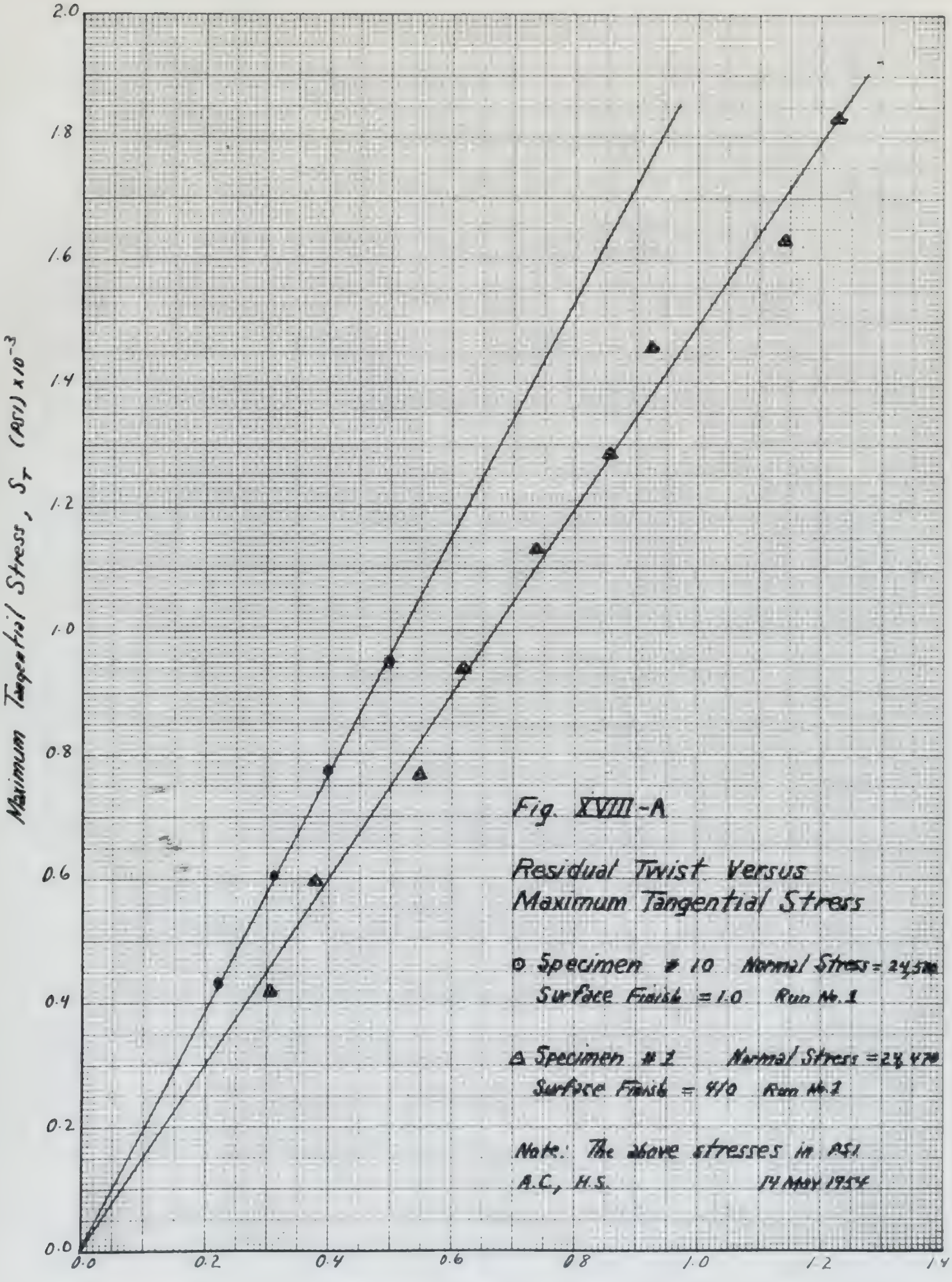


Fig. XVIII-A

Residual Twist Versus Maximum Tangential Stress

Note: The above stresses in PSI

A.C., H.S. 14 May 1954

DISCUSSION OF RESULTS

Part 1. Test on Specimens 1 - 10

These tests were, on a qualitative basis, surprisingly consistent. Figures I to X show that on all specimens where more than one run was made, the residual twist for a given maximum tangential stress decreased as the normal stress increased. This is further substantiated by Figures XV and XVI. Figure XV shows the results for two specimens with the same surface finish (1.0), one specimen being beveled at the ends and the other unbeveled. Figure XVI compares four specimens having a 4/0 finish. These figures show the same effect of normal stress.

Figures XVII and XVIII-A show the effect when surface finish is varied while normal stress is maintained constant. Figure XVII compares three different finishes at approximately the same normal stress (70,000 psi). It is seen that, for a given value of maximum tangential stress, the residual twist varies with the roughness (i.e., the rougher the surface, the greater the residual twist). Figure XVIII-A, which compares two specimens with surface finishes of 4/0 and 1.0 at a low normal stress, (approximately 24,500 psi), shows that the residual twist increases with the surface smoothness.

If the above results are valid, there should be some value of normal stress at which the residual twist for a given maximum tangential stress for both 4/0 and 1.0 finish are the same. This

is borne out by comparing the curve in Figure XV for specimen # 10, surface finish 1.0, Run 4, normal stress 48,800 psi with curves in Figure XVI for specimen #3, surface finish 4/0, Runs 1 and 2 with normal stresses of 44,600 and 52,100 psi respectively. This comparison shows that there is a transition range for normal stress in which the effect of surface finish on residual twist reverses (i.e., smooth finishes give more twist than the rough finishes below this range and less twist above it).

In evaluating our results some information upon the accuracy of the readings is in order. One scale unit of the optic vernier in the microscope eyepiece represents 0.490 minutes of twist for the first six specimens and 0.457 minutes for the succeeding specimens. The reason for the difference is that a 7-1/2 inch indicator arm was substituted for the original 7.0 inch indicator. Readings were estimated to the nearest tenth of a scale unit. This allowed an accuracy of plus or minus one tenth scale unit or 0.0490 and 0.046 minutes of twist respectively.

Part 2. Tests on Specimens 12 - 14.

Comparing Figures XII through XIV, it is noted that the results for these runs were not as consistent as those in Part 1. Figure XII does not show the residual twist decreasing with increasing normal stress as was observed in Part 1. Figures XIII and XIV show this trend only slightly.

Figure XVIII which compares the effects of surface finish for

a normal stress of 21,830 psi shows that the 4/0 finish gives more residual twist than the 1.0 finish for any given maximum tangential stress. This is in accord with the results in Part 1, as shown in Figure XVIII-A.

As in Part 1, a brief explanation of the accuracy of the readings will be helpful in evaluating these results.

In this series of tests, as noted in the Procedure, two indicator arms were used. For one arm the accuracy of reading was plus or minus one tenth scale unit. For two arms, therefore, our accuracy was decreased to plus or minus two tenths of a scale unit. This means our readings of twist are accurate to only plus or minus 0.092 minutes of twist. Thus, in Figures XII through XIV the accuracy of the comparison is the sum of the accuracy of each curve or in other words the comparison can be accurate to only plus or minus 0.184 minutes of residual twist. This possible range of error is much greater than any of the observed differences shown in the curves, hence it is felt that this data cannot validly be used to determine the effect of normal stress on residual twist. Hindsight is admittedly better than foresight. It would have been better to have used a much wider range of normal loads. However, our apparatus would not permit this and time limitations prevented assembling new apparatus. Another solution would be the use of a more powerful microscope with a finer optic vernier scale.

Figure XVIII is not rendered invalid by the above limits on

the accuracy of the readings since for the larger values of maximum tangential stress the difference in residual twist due to surface finish is greater than 0.184 minutes of residual twist and hence, these results are valid for qualitative analysis.

Part 3. Authors' Theory on Residual Twist in Interfaces

The results of this work clearly show that metal interfaces do exhibit elastic behavior. The following explanation is offered.

It is well known that all metal surfaces regardless of the degree of surface finish are covered with small irregularities called asperities. These will vary with the degree of surface finish from jagged peaks to rolling downs. It is believed that these asperities act as cantilevers.

The calculations in Appendix C show that if the asperities are assumed to act as bulk metal, then to account for the residual twist would require an asperity height which is much too large. However, if the asperities are assumed to be cantilevers, an asperity height can be calculated that is of the same order of magnitude as that measured by a profilometer.

Further, the cantilever theory offers an explanation for the transition range mentioned earlier in Part 1.

Bowden and Tabor (4) state that the "real contact area" is directly proportional to the normal load and is independent of surface finish. It is known that the number of asperities will increase and the height will decrease with the degree of surface

finish. Therefore, for any normal load, a smoother surface finish will have more asperities in contact, but the asperity height will be less than that for the rougher surface.

The height of asperity also depends upon the normal load since the tips of the asperities will flow plastically to accomodate this load. There is a limit to this plastic flow in the asperities, apparently, for Bowden and Tabor (4) show photographically (Plate II) that even when the normal load is great enough to cause the bulk metal to flow plastically, the asperities retain their identity.

Thus, the twist for a given maximum tangential stress is dependent on the number of asperities and the normal load, since this load affects the asperity height. For low normal stresses, apparently the number of cantilevers offsets the effect of the shorter height in smooth surfaces as shown in Figures XVIII and XVIII-A. As the normal stress is increased and the limit of plastic flow is reached in the asperities, the height differential becomes the controlling factor affecting twist so that the asperities on the smoother surface display less twist than those on rougher surfaces.

CONCLUSIONS AND RECOMMENDATIONS

A. It is concluded that:

1. The asperities in steel interfaces contribute materially to the elastic twist.
2. The elastic twist due to an interface, for a given value of maximum tangential stress, decreases with increasing normal stress.
3. The effect of surface finish is most interestingly affected by normal stress in that a transition range exists for normal stress. Below this range the smoother finish gives the greater elastic twist, whereas above this range, the rougher finish gives the greater twist.
4. All the above effects are explained if the asperities are considered to behave as cantilevers.

B. It is recommended that:

1. Further investigations be made of the transition range for degrees of surface finish other than 1.0 and 4/0.
2. Annular specimens be tested over a greater range of normal stresses.
3. Metals of different physical characteristics be studied.

APPENDIX A

Details of Procedure

DETAILS OF PROCEDURE

As in the section on Procedure, the discussion will be separated into two parts corresponding to the two experimental methods used.

In the first, practical considerations prevented location of the indicator arm exactly at the interface. Consequently the measured twist consisted of two parts: 1, that contributed by the interface; 2, that contributed by twist of the specimen between the interface and the angle indicator.

Since the angle indicator was attached to the specimen by pointed set screws, the distance between it and the interface could be determined with a high degree of accuracy. However, to determine the twist of the specimen required deriving an expression for computing the twist in that part of the specimen that was bevelled. The derivation is given below.

Refer to Figure XX. Consider a plane perpendicular to the axis of specimen intersecting the specimen at a distance "x" from the interface. The shear stress existing in this intersection is given by (4)

$$\text{Shear stress} = Gr\frac{d\phi}{dx} = \sigma \qquad (1)$$

G = modulus of elasticity in shear (for our specimen $G = 12 \times 10^6$ pounds per square inch). $\frac{d\phi}{dx}$ = angle of twist per unit length. Considering an element of area, dA

$$dA = 2\pi r\,dr \tag{2}$$

The moment of the shearing force about the axis of the specimen for this element of area is $\sigma r\,dA$

The total moment about the axis of the specimen must equal the effective torque in the specimen. As explained in the Procedure, the effective torque equals one half the applied torque. Therefore,

$$T_B = 1/2\ T_A = \int_0^r \sigma\, r\,dA \tag{3}$$

Substituting equations (1) and (2) into equation (3)

$$T_A = \int_0^r 4\pi G\ \frac{d\phi}{dx}\ r^3\,dr \tag{4}$$

In any given plane perpendicular to the axis of the specimen $d\phi/dx$ is independent of r. Therefore equation (4) can be integrated.

$$T_A = \pi G \frac{d\phi}{dx}\ r^4 \tag{5}$$

Rearranging gives

$$d\phi = \frac{T_A\,dx}{\pi\ Gr^4} \tag{6}$$

It is evident in Figure XX that r is related to x by

$$r = r_0 + \frac{R - r}{\ell}\,x = r_0 + kx \tag{7}$$

r_0 = radius at $x = 0$ (i.e., $r_0 = d/2$).

R = radius at $x = \ell$ (i.e., $R = D/2$).

Substituting equation (7) into equation (6)

$$d\phi = \frac{T_A \; dx}{\pi G(r_0 + kx)^4} \qquad (8)$$

Equation (8) can be integrated for the angle of twist over the length ℓ.

$$\phi_\ell = \int_0^\ell d\phi = \int_0^\ell \frac{T_A \; dx}{\pi G(r_0 + kx)^4} = \frac{T_A}{3\pi Gk} \; \left(\frac{1}{r}\right)^3 - \left(\frac{1}{r_0 + \ell k}\right)^3 \qquad (9)$$

this can be simplified by substituting

$$k = \frac{R - r_0}{\ell}$$

$$r_0 = d/2$$

$$R = D/2$$

Performing this substitution gives

$$\phi_\ell = \frac{8 \; T_A}{3\pi Gk} \left[\left(\frac{1}{d}\right)^3 - \left(\frac{1}{D}\right)^3 \right] \qquad (10)$$

this can be expressed more conveniently for our purposes by making the following substitutions

Ψ_ℓ = twist in minutes = $\frac{(60)(180)}{\pi} \phi_\ell$

Q_A = applied torque in gram-inches = $\frac{1000}{2.205} \; T_A$

$G = 12 \times 10^6$ lb/in^2

This gives

$$\Psi_\ell = \frac{(60)(180)}{\pi} \times \frac{8}{3\pi} \times \frac{2.205}{1000} \; Q_A \; (1/k) \left[(1/d)^3 - (1/D)^3 \right]$$

$$\text{or, } \Psi_\ell = 5.360 \times 10^{-7} \; (1/k) \left[(1/d)^3 - (1/D)^3 \right] Q_A \qquad (11)$$

The twist in the specimen between $x = \ell$ and $x = \ell'$ (i.e., between the upper end of the bevel and the indicator arm) can be easily calculated (4).

$$\phi_{\ell'} = \frac{32\ T_x}{\pi\ GD^4}\ \ell' \tag{12}$$

This can be expressed in the same convenient form as equation (11)

$$\psi_{\ell'} = 3.205 \times 10^{-6}\ \frac{\ell'}{D^4}\ Q_A \tag{13}$$

Thus the total twist in the specimen between the interface and the indicator arm is

$$\psi_T = \psi_\ell + \psi_{\ell'} \quad (\text{Min. of Arc}) \tag{14}$$

The total twist as computed above is subtracted from the observed twist to obtain what we have called "residual twist" and designated ψ_R.

$$\psi_R = \psi_0 - \psi_T \tag{15}$$

With the annular specimens, it was possible to use a control specimen as described in the procedure. This permitted us to obtain a value for twist per gram inch of torque per inch of specimen length under actual experimental conditions. For subsequent runs it was possible to compare the observed twist with what the twist would have been if there had been no interface. As before the difference was called "residual twist".

Fig. XIX
Interface Testing Apparatus
Scheme - One (NOT TO SCALE)

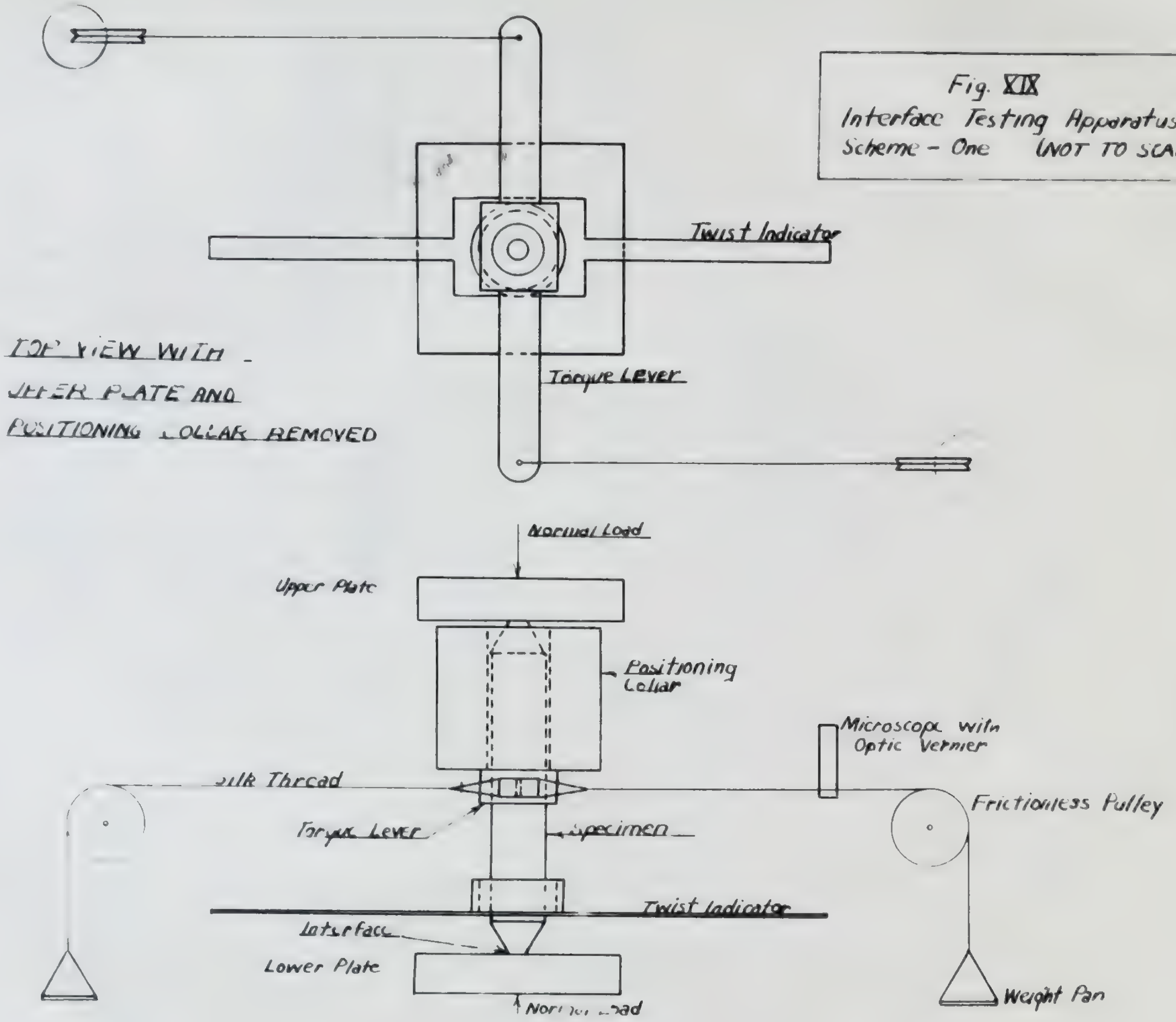


Fig. XX
Nomenclature Used In
Computations (From Fig. XIX)

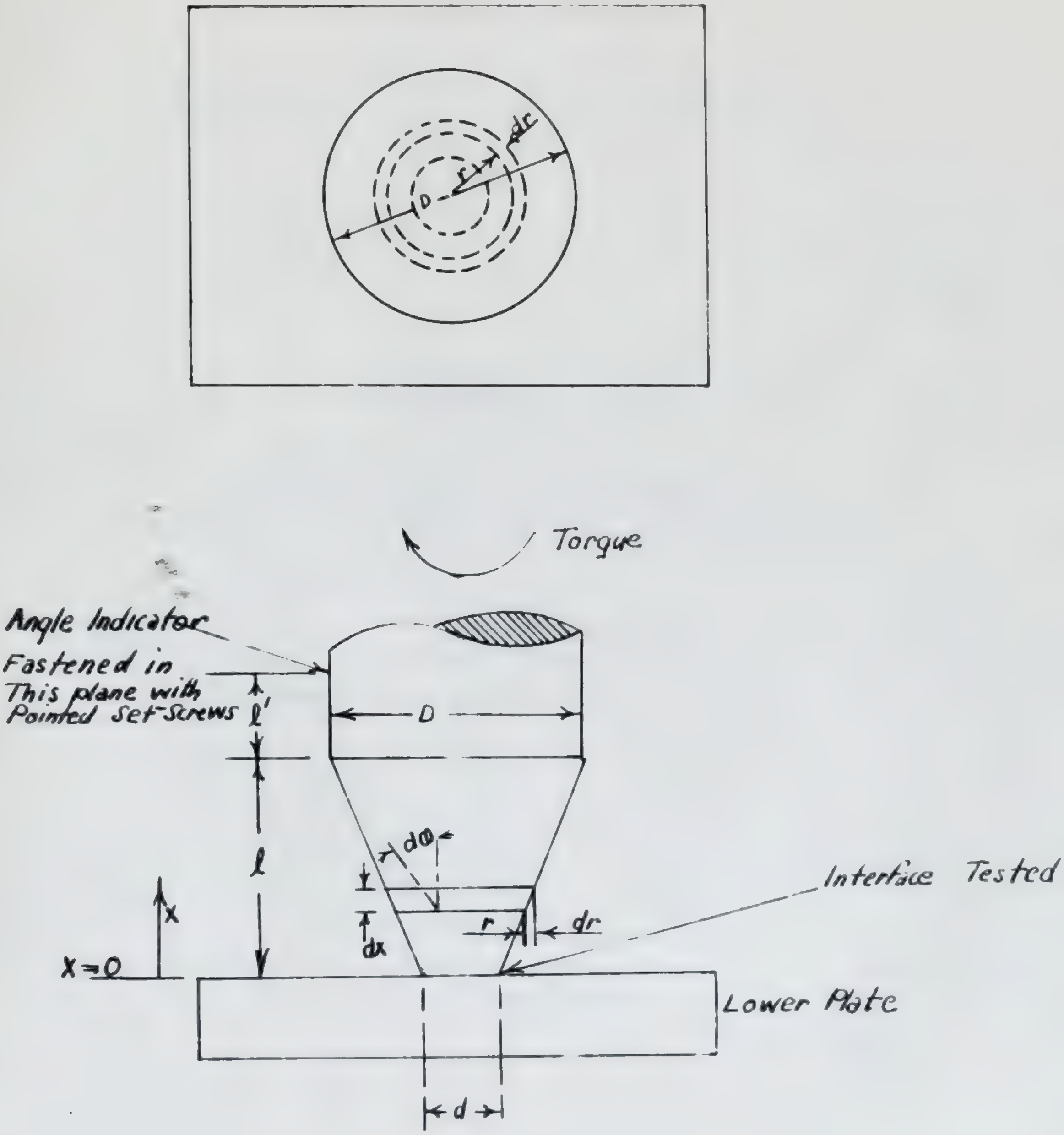
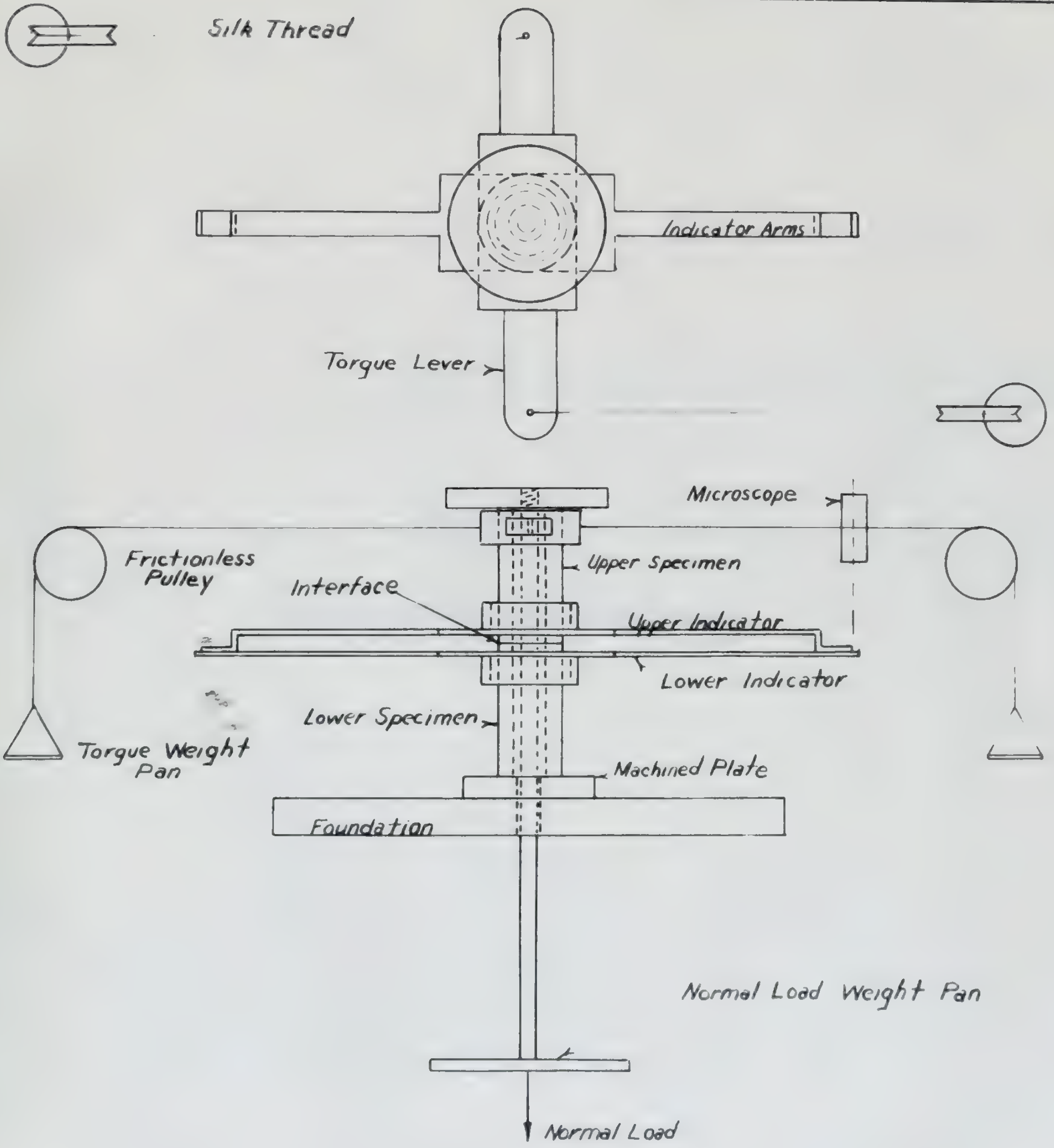


Fig XXI
Interface Testing Apparatus
Scheme - Two (Not to Scale)



APPENDIX B

Summary of Data and Calculations

SPECIMEN # 1

$d = 0''.125 = D$

$\ell = 0''.000$

$\ell' = 0''.096$

$F = 0.490$

$\psi_\ell = 0.000$

$\psi_{\ell'} = 3.205 \times 10^{-6} \frac{\ell'}{D^4} Q_A$

$\psi_{\ell'} = 1.269 \times 10^{-3} Q_A$

$\psi_T = \psi_\ell + \psi_{\ell'}$

$\psi_T = 1.269 \times 10^{-3} Q_A$

$\psi_0 = F \times \psi_S = 0.49 \psi_S$

$\psi_R = \psi_0 - \psi_T$

$S_T = 5.610 \times 10^{-3} \frac{Q_A}{d^3}$

$S_T = 2.88 Q_A$

$S_N = \frac{F_N}{\pi/4 \, d}$

Surface Finish: 4/0

$F_N = 300$

$S_N = 24,500$

Q_A	S_T	ψ_T	ψ_0	ψ_R
145.86	418	0.185	0.490	.305
205.86	593	0.261	0.637	.376
265.86	765	0.337	0.882	.545
325.86	938	0.413	1.030	.617
385.86	1130	0.489	1.225	.736
445.86	1285	0.565	1.421	.856
505.86	1459	0.645	1.568	.923
565.86	1630	0.718	1.861	1.143
625.86	1830	0.781	2.010	1.229
685.86	1978	0.858	2.205	1.347
745.86			—	

SPECIMEN # 2

$d = 0''.08045$
$D = 0''.1248$
$\ell = 0''.08045$
$\ell' = 0''.000$
$F = 0.490$

$F_N = 277.6$

$S_N = 53{,}750$

$$\psi_\ell = \frac{5.360 \times 10^{-7}}{k}\left[\left(\frac{1}{d}\right)^3 - \left(\frac{1}{D}\right)^3\right] Q_A$$

$\psi_{\ell'} = 0''.000$

$\psi_T = \psi_\ell + \psi_{\ell'}$

$\psi_T = 1.229 \times 10^{-3}\, Q_A$

$\psi_o = F \times \psi_S = 0.49\, \psi_S$

$\psi_R = \psi_o - \psi_T$

$S_T = 5.610 \times 10^{-3}\, \frac{Q_A}{d^3}$

$S_T = 9.83\, Q_A$

$S_N = \frac{F_N}{\pi/4\, d^2}$

Surface Finish: 2/0

Q_A	S_T	ψ_T	ψ_O	ψ_R
145.86	1432	0.179	0.784	.605
217.86	2140	0.267	1.078	.811
337.86	3320	0.414	2.058	1.644
398.34	3918	0.490	2.450	1.960
458.82	4500	0.563	2.842	2.270
519.30			—	

SPECIMEN # 4

$d = 0''.081$
$D = 0''.125$
$\ell = 0''.00782$
$\ell' = 0''.1088$
$F = 0.490$

$$\psi_\ell = \frac{5.360 \times 10^{-7}}{k} \left[\left(\frac{1}{d}\right)^3 - \left(\frac{1}{D}\right)^3 \right] Q_A$$

$$= 0.267 \times 10^{-3}\ Q_A$$

$$\psi_{\ell'} = 3.205 \times 10^{-6} \frac{\ell'}{D^4}\ Q_A$$

$$= 1.430 \times 10^{-3}\ Q_A$$

$$\psi_T = \psi_\ell + \psi_{\ell'}$$

$$= 1.697 \times 10^{-3}\ Q_A$$

$$\psi_0 = F \times \psi_S = 0.490\ \psi_S$$

$$S_T = 5.610 \times 10^{-3}\ \frac{Q_A}{d^3}$$

$$S_T = 10.57\ Q_A$$

$$S_N = \frac{F_N}{\pi/4\ d^2}$$

Surface Finish: 4/0

$F_N = 300$
$S_N = 58,200$

Q_A	S_T	ψ_T	ψ_0	ψ_R
145.86	1541	.248	.496	.242
205.86	2175	.349	.687	.338
265.86	2810	.451	.952	.481
325.86	3440	.552	1.079	.527
385.86	4080	.654	1.372	.718
445.86	4710	.756	1.520	.764
505.86			—	

SPECIMEN # 3

$d = 0''.082$

$D = 0''.125$

$\ell = 0''.1132$

$\ell' = 0''.0756$

$F = 0.490$

$$\psi_\ell = \frac{5.360 \times 10^{-7}}{k} \left[\left(\frac{1}{d}\right)^3 - \left(\frac{1}{D}\right)^3 \right] Q_A$$

$$\psi_\ell = 3.680 \times 10^{-3}\ Q_A$$

$$\psi_{\ell'} = 3.205 \times 10^{-6}\ \frac{\ell'}{D^4}\ Q_A$$

$$\psi_{\ell'} = 0.994 \times 10^{-3}\ Q_A$$

$$\psi_T = \psi_\ell + \psi_{\ell'}$$

$$\psi_T = 4.674 \times 10^{-3}\ Q_A$$

$$\psi_0 = F \times \psi_S = 0.49\ \psi_S$$

$$\psi_R = \psi_0 - \psi_T$$

$$S_T = 5.610 \times 10^{-3}\ \frac{Q_A}{d^3}$$

$$S_T = 10.2\ Q_A$$

$$S_N = \frac{F_N}{\pi/4\ d^2}$$

Surface Finish = 4/0

			RUN # 1 $F_N = 300$ $S_N = 44,600$		RUN # 2 $F_N = 350$ $S_N = 52,100$	
Q_A	S_T	ψ_T	ψ_0	ψ_R	ψ_0	ψ_R
145.86	1488	.631	0.980	.299	1.029	.348
205.86	2100	.962	1.470	.508	1.470	.508
265.86	2710	1.241	1.960	.719	1.911	.670
325.86	3320	1.520	2.450	.930	2.890	.880
385.86	3930	1.800	—	—	3.380	1.090
445.86					—	—

$d = 0''.0776$

$D = 0''.125$

$\ell = 0''.0562$

$\ell' = 0''.0776$

$F = 0.490$

$$\psi_{\ell} = \frac{5.360 \times 10^{-7}}{k}\left[\left(\frac{1}{d}\right)^3 - \left(\frac{1}{D}\right)^3\right]Q_A$$

$$= 2.070 \times 10^{-3} Q_A$$

$$\psi_{\ell'} = 3.205 \times 10^{-6}\ \frac{\ell'}{D^4}\ Q_A$$

$$= 1.020 \times 10^{-3}\ Q_A$$

$$\psi_T = \psi_{\ell} + \psi_{\ell'} = 3.090 \times 10^{-3} Q_A$$

$$\psi_o = F \times \psi_S = 0.490\ \psi_S$$

$$S_T = 5.610 \times 10^{-3}\ \frac{Q_A}{d^3}$$

$$S_T = 11.042\ Q_A$$

$$S_N = \frac{F_N}{\pi/4\ d^2}$$

Surface Finish: Ø

			RUN # 1		RUN # 2		RUN # 3	
			$F_N = 300$ $S_N = 62{,}800$		$F_N = 350$ $S_N = 73{,}300$		$F_N = 373$ $S_N = 83{,}800$	
Q_A	S_T	ψ_T	ψ_o	ψ_R	ψ_o	ψ_R	ψ_o	ψ_R
150	1656	.464	.784	.320	.784	.320	.784	.320
210	2319	.649	1.029	.380	1.029	.380	1.029	.380
270	2981	.834	1.470	.636	1.470	.636	1.470	.636
330	3644	1.020	1.911	.891	1.911	.891	1.813	.793
390	4307	1.205	2.303	1.098	2.156	.951	2.009	.804
450	4969	1.390	—		2.450	1.060	2.401	1.010
510	5632	1.576			2.842	1.266	2.744	1.168
570	6294	1.761			3.185	1.424	3.081	1.326
630					—	—	—	—

SPECIMEN # 5 (continued)

			RUN # 4		RUN # 5		RUN # 6		RUN # 7	
			F_N = 400 S_N = 83,800		F_N = 450 S_N = 93,400		F_N = 500 S_N = 105,000		F_N = 550 S_N = 115,200	
S_A	S_T	ψ_T	ψ_o	ψ_R	ψ_o	ψ_R	ψ_o	ψ_R	ψ_o	ψ_R
150	1656	.464	.784	.320	.784	.320	.784	.320	.588	.124
210	2319	.649	1.029	.380	1.029	.380	1.029	.380	.931	.282
270	2981	.834	1.470	.636	1.372	.538	1.323	.489	1.225	.391
330	3644	1.020	1.715	.695	1.617	.597	1.568	.548	1.470	.450
390	4307	1.205	1.960	.755	1.911	.706	1.960	.755	1.813	.608
450	4964	1.390	2.352	.962	2.303	.913	2.205	.815	2.009	.619
510	5632	1.576	2.656	1.080	2.607	1.031	2.450	.874	2.450	.874
570	6294	1.761	2,940	1.179	2.940	1.179	2.842	1.081	—	—
630					—	—	—	—		

SPECIMEN # 6

$d = 0''.0783$
$D = 0''.125$
$l = 0''.0562$
$l' = 0''.0783$
$F = 0.490$

$$\psi_l = \frac{5.360 \times 10^{-7}}{k}\left[\left(\frac{l}{d}\right)^3 - \left(\frac{l}{D}\right)^3\right] Q_A$$

$$= 2.023 \times 10^{-3}\ Q_A$$

$$\psi_{l'} = 3.205 \times 10^{-3}\ \frac{l'}{D^4}\ Q_A$$

$$= 1.028 \times 10^{-3}\ Q_A$$

$$\psi_T = \psi_l + \psi_{l'} = 3.051\ Q_A$$

$$\psi_0 = F \times \psi_S = 0.490\ \psi_S$$

$$S_T = 5.610 \times 10^{-3}\ \frac{Q_A}{d^3}$$

$$S_T = 11.68\ Q_A$$

$$S_N = \frac{F_N}{\pi/4 d^2}$$

Surface Finish: as machined

			RUN # 1		RUN # 2		RUN # 3		RUN # 4	
			F_N = 300		F_N = 325		F_N = 350		F_N = 375	
			S_N = 62,500		S_N = 67,450		S_N = 72,600		S_N = 77,800	
Q_A	S_T	ψ_T	ψ_0	ψ_R	ψ_0	ψ_R	ψ_0	ψ_R	ψ_0	ψ_R
150	1752	.457	.784	.327	.784	.327	.784	.327	.833	.376
210	2454	.640	1.078	.438	1.078	.438	1.078	.438	1.127	.487
270	3156	.823	1.470	.647	1.470	.647	1.470	.647	1.421	.548
330	3858	1.006	1.960	.954	1.813	.807	1.813	.807	1.813	.807
390	4560	1.189	2.499	1.310	2.158	.967	2.107	.918	2.156	.967
450	5262	1.372	—		—	—	2.450	1.078	2.450	1.078
510	5964	1.555			—		—		2.842	1.287
570									—	

SPECIMEN # 6 (continued)

			RUN # 5		**RUN # 6**		**RUN # 7**		**RUN # 8**		**RUN # 9**		**RUN # 10**	
			F_N = 400		F_N = 425		F_N = 440		F_N = 454		F_N = 460		F_N = 525	
			S_N = 83,000		S_N = 88,200		S_N = 91,300		S_N = 94,150		S_N = 95,500		S_N = 109,000	
C_A	S_T	ψ_T	ψ_O	ψ_R	ψ_O	ψ_R	ψ_O	ψ_R	ψ_O	ψ_R	ψ_O	ψ_R	ψ_O	ψ_R
150	1,752	.457	.784	.327	.686	.229	.637	.180	.784	.327	.784	.327	.735	.278
210	2,454	.640	1.127	.487	1.127	.487	1.078	.438	1.127	.487	1.024	.389	1.372	.549
270	3,156	.823	1.421	.598	—		1.372	.549	1.421	.598	1.421	.588	1.960	.771
330	3,858	1.006	1.764	.758			1.617	.611	—		1.764	.758	2.303	.931
390	4,560	1.189	2.107	.918			—				2.009	.820	2.464	1.091
450	5,262	1.372	2.401	1.029							2.352	.980	—	
510	5,964	1.555	2.744	1.189							—			
570	6,666	1.738	3.087	1.349										
630			—											

SPECIMEN # 7

d = 0".0949
D = 0".125
ℓ = 0".0410
ℓ' = 0".3598
F = 0.457

$$\psi_\ell = \frac{5.360 \times 10^{-7}}{k}\left[\left(\frac{1}{d}\right)^3 - \left(\frac{1}{D}\right)^3\right] Q_A$$

$$= 0.900 \times 10^{-3}\, Q_A$$

$$\psi_{\ell'} = 3.205 \times 10^{-3}\, \frac{\ell'}{D^4}\, Q_A$$

$$= 4.720 \times 10^{-3}\, Q_A$$

$$\psi_T = \psi_\ell + \psi_{\ell'} = 5.700 \times 10^{-3}\, Q_A$$

$$\psi_0 = F \times \psi_S = 0.457\ \psi_S$$

$$S_T = 5.610 \times 10^{-3}\, \frac{Q_A}{d^3}$$

$$S_T = 6.569\, Q_A$$

Surface Finish #3

			RUN # 1		RUN # 2		RUN # 3	
			F_N = 400		F_N = 450		F_N = 500	
			S_N = 56,600		S_N = 63,700		S_N = 70,800	
Q_A	S_T	ψ_T	ψ_0	ψ_R	ψ_0	ψ_R	ψ_0	ψ_R
150	985	.855	1.280	.425	1.280	.425	1.280	.425
180	1182	1.026	——	——	1.462	.436	1.508	.482
210	1380	1.197	1.782	.585	1.737	.540	1.737	.540
240	1577	1.368	——	——	1.919	.551	1.964	.596
270	1774	1.539	2.285	.746	2.239	.700	2.239	.700
300	1971	1.710	——	——	2.468	.758	—	
330	2168	1.881	2.788	.907	2.696	.815		
360	2365	2.052			2.970	.918		
390	2562	2.223	—		3.245	1.022		
420					—			

SPECIMEN # 8

d = 0".0958
D = 0".125
ℓ = 0".03105
ℓ' = 0".3660
F = 0.457

$$\psi_\ell = \frac{5.360 \times 10^{-7}}{k}\left[\left(\frac{1}{d}\right)^3 - \left(\frac{1}{D}\right)^3\right] Q_A$$

$$= 0.715 \times 10^{-3}\ Q_A$$

$$\psi_{\ell'} = 3.205 \times 10^{-6}\ \frac{\ell'}{D^4}\ Q_A$$

$$= 4.800 \times 10^{-3}\ Q_A$$

$$\psi_T = \psi_\ell + \psi_{\ell'} = 5.515 \times 10^{-3}\ Q_A$$

$$\psi_O = F \times \psi_S = 0.457\ \psi_S$$

$$S_T = 5.610 \times 10^{-3}\ \frac{Q_A}{d^3}$$

$$S_T = 6.3898\ Q_A$$

$$S_N = \frac{F_N}{\pi/4\ d^2}$$

Surface finish: 1.0

			RUN # 1		RUN # 2		RUN # 3	
			F_N = 350		F_N = 400		F_N = 500	
			S_N = 48,400		S_N = 55,400		S_N = 69,000	
Q_A	S_T	ψ_T	ψ_O	ψ_R	ψ_O	ψ_R	ψ_O	ψ_R
150	958	.827	1.005	.178	1.005	.178	1.005	.178
210	1342	1.158	1.462	.304	1.508	.356	1.462	.304
270	1725	1.489	1.965	.476	1.965	.476	1.874	.385
330	2109	1.820	2.468	.648	2.422	.602	2.331	.511
390	2492	2.151	—		2.879	.728	2.788	.637
450	2875	2.482			3.336	.854	3.199	.717
510	3259	2.813			3.793	.980	3.656	.843
570					—		—	

(continued on next page)

SPECIMEN # 8 (continued)

			RUN # 4		RUN # 5		RUN # 6		RUN # 7	
			F_N = 550		F_N = 600		F_N = 800		F_N = 900	
			S_N = 76,100		S_N = 83,100		S_N = 110,900		S_N = 124,800	
S_A	S_T	ψ_T	ψ_0	ψ_R	ψ_0	ψ_R	ψ_0	ψ_R	ψ_0	ψ_R
150	958	.827	1.005	.178	1.005	.178	1.005	.178	1.005	.176
210	1342	1.158	1.417	.259	1.417	.259	1.417	.259	1.371	.213
270	1725	1.489	1.828	.339	1.874	.385	1.782	.293	1.737	.248
330	2109	1.820	2.285	.465	2.285	.465	2.194	.374	—	
390	2492	2.151	2.696	.595	2.696	.545	—	—		
450	2875	2.482	3.153	.671	—					
510	3259	2.813	3.610	.797						
570			—							

SPECIMEN # 9

d = 0".0949
D = 0".125
ℓ = 0".0421
ℓ' = 0".3629
F = 0.457

$$\Psi_\ell = \frac{5.36 \times 10^{-7}}{k}\left[\left(\frac{1}{d}\right)^3 - \left(\frac{1}{D}\right)^3\right] Q_A$$

$$= 0.989 \times 10^{-3}\, Q_A$$

$$\Psi_{\ell'} = 3.205 \times 10^{-6}\, \frac{\ell'}{D^4}\, Q_A$$

$$= 4.760 \times 10^{-3}\, Q_A$$

$$\Psi_T = \Psi_\ell + \Psi_{\ell'} = 5.749 \times 10^{-3}\, Q_A$$

$$\Psi_0 = F \times \Psi_S = 0.457\, \Psi_S$$

$$S_T = 5.610 \times 10^{-3}\, \frac{Q_A}{d^3}$$

$$S_T = 6.569\, Q_A$$

Surface Finish: 4/0

			RUN # 1		RUN # 2		RUN # 3	
			F_N = 350		F_N = 450		F_N = 500	
			S_N = 49,500		S_N = 63,700		S_N = 70,800	
Q_A	S_T	Ψ_T	Ψ_0	Ψ_R	Ψ_0	Ψ_R	Ψ_0	Ψ_R
150	985	.862	1.005	.143	.960	.098	.914	.052
210	1380	1.207	1.371	.164	1.418	.211	——	——
270	1774	1.552	1.919	.367	1.828	.276	1.828	.276
330	2168	1.898	2.468	.570	2.285	.387	——	——
390	2562	2.242	3.016	.770	2.742	.500	2.742	.500
450	2956	2.587	—		3.199	.612	——	——
510	3350	2.932			3.656	.724	3.656	.724
570	3745	3.277			4.113	.836	——	——
630	4134	3.622			4.570	.948	4.570	.948
690	4533	3.967			5.027	1.060	——	——
750	4927	4.312			5.484	1.172	5.438	1.126
810	5321	4.657			5.941	1.284	—	
870	5715	5.002			6.398	1.396		
930					—			

SPECIMEN # 10

$d = D = 0''.125$
$\ell = 0''.000$
$\ell' = 0''.3519$
$F = 0.457$

$\psi_\ell = 0.000$

$\psi_{\ell'} = 3.205 \times 10^{-6} \frac{\ell'}{D^4} Q_A$

$= 4.613 \times 10^{-3} Q_A$

$\psi_T = \psi_\ell + \psi_{\ell'} = 4.613 \times 10^{-3} Q_A$

$\psi_o = F \times \psi_s = 0.457 \psi_s$

$S_T = 5.61 \times 10^{-3} \frac{Q_A}{d^3}$

$S_T = 2.8723 Q_A$

$S_N = \frac{F_N}{\pi/4 \, d^2}$

Surface Finish: 1.0

			RUN # 1		RUN # 2		RUN # 3	
			$F_N = 300$		$F_N = 400$		$F_N = 500$	
			$S_N = 24,500$		$S_N = 32,600$		$S_N = 40,800$	
Q_A	S_T	ψ_T	ψ_O	ψ_R	ψ_O	ψ_R	ψ_O	ψ_R
150	431	.692	.914	.222	.914	.222	.868	.176
210	603	.969	1.280	.311	1.234	.265	——	——
270	776	1.246	1.645	.399	1.554	.308	1.461	.216
330	948	1.522	2.011	.489	1.874	.352	——	——
390	1120	1.794	—		2.194	.395	2.102	.403
450	1293	2.076			2.559	.483	——	——
510	1465	2.353			2.833	.480	2.788	.435
570	1637	2.629			3.153	.524	——	——
630	1810	2.906			3.519	.613	3.473	.567
690	1982	3.183			3.893	.656	——	——
750	2154	3.460			4.204	.744	4.204	.744
810	2327	3.737			4.520	.833	——	——
870	2499	4.013			—		—	
930	2671	4.290						
1050	3016	4.844						
1170	3316	5.397						
1290	3705	5.951						
1470								

(continued on next page)

SPECIMEN # 10 (Continued)

			RUN # 4		**RUN # 5**		**RUN # 6**	
			F_N = 600		F_N = 700		F_N = 900	
			S_N = 48,900		S_N = 57,030		S_N = 73,400	
C_A	S_T	ψ_T	ψ_0	ψ_R	ψ_0	ψ_R	ψ_0	ψ_R
150	431	.692	.823	.131	.823	.131	.777	.085
210	603	.969	1.097	.128	1.417	.171	1.051	.082
270	776	1.246	1.417	.171	2.102	.303	1.371	.125
330	948	1.522	1.828	.306	2.742	.389	1.782	.260
390	1120	1.794	2.102	.303	3.482	.522	2.057	.258
450	1293	2.076	2.468	.392	——	——	——	——
510	1465	2.353	——	——	4.067	.607	2.696	.343
570	1637	2.629	3.153	.524	——	——	——	——
630	1810	2.906	——	——	4.707	.694	3.290	.384
690	1982	3.183	3.747	.564	——	——	——	——
750	2154	3.460	——	——	5.347	.780	3.976	.516
810	2327	3.737	4.387	.650	——	——	——	——
870	2499	4.013	——	——	—		4.570	.557
930	2671	4.290	5.188	.828	——	——	——	——
990	2844	4.567					5.210	.643
1050	3016	4.844	5.667	.823	——	——	——	——
1170	3316	5.397	6.398	1.001	——	——	——	——
1290	3705	5.951	6.992	1.041	——	——	——	——
1470			—	——	——	——	——	——

SPECIMEN # 11

(Control Specimen For Runs on Annuli)

$D_1 = 0.171$

$D_o = 0.251$

$\ell = 0''.5705$

$F = 0.457$

$\psi_o = F \times \psi_s$

$\psi_o = 0.457\, \psi_s$

$$S_T = 1.121 \times 10^{-2} \frac{D_o}{D_o^4 - D_1^4} Q_A$$

$$S_T = 0.903\, Q_A$$

$$S_N = \frac{F_N}{\pi/4\,(D_o^2 - D_1^2)}$$

RUN # 1 RUN # 2

$F_N = 150$ $F_N = 310$

$S_N = 10{,}563$ $S_N = 21{,}830$

Q_A	S_T	ψ_0	ψ_0
150	135	.137	.091
300	271	.229	.274
450	406	.366	.366
600	542	.503	.548
750	677	—	.686
900	813		.713
1050	948		.914
1200			—

From the plot of this data, Figure XI, it was found this specimen deflected

$$\frac{0.914}{1050} = 8.705 \times 10^{-4}$$

minutes per gram-inch of torque. Since the length between our indicator arms, ℓ, is $0''.5705$, this gives an average deflection of

$$\frac{8.705 \times 10^{-4}}{0.5705} = 1.5258 \times 10^{-3}$$

minutes per gram-inch of torque per inch of length.

This number enables us to compare the deflections obtained in Runs on specimens, 12, 13, and 14 with the deflection for a solid specimen (i.e., no interface).

SPECIMEN # 12

$D_i = 0''.171$
$D_o = 0''.251$
$l = 0''.5702$
$F = 0.457$

$\psi_C = 1.5258 \times 10^{-3} \, l \, Q_A$

$= 0.8700 \times 10^{-3} \, Q_A$

$\psi_o = F \times \psi_S$

$= 0.457 \, \psi_S$

$S_T = 1.121 \times 10^{-2} \dfrac{D_o}{D_o^4 - D_i^4} Q_A$

$S_N = \dfrac{F_N}{\pi/4 \, (D_o^2 - D_i^2)}$

Surface finish:
as machined 1380

			RUN # 1		**RUN # 2**		**RUN # 3**		**RUN # 4**	
			$F_N = 150$		$F_N = 202$		$F_N = 250$		$F_N = 310$	
			$S_N = 10,563$		$S_N = 14,225$		$S_N = 17,605$		$S_N = 21,830$	
Q_A	S_T	ψ_C	ψ_O	ψ_R	ψ_O	ψ_R	ψ_O	ψ_R	ψ_O	ψ_R
150	135	.131	.229	.098	.183	.052	.137	.006	.229	.098
300	271	.261	.457	.196	.411	.150	.320	.059	.457	.196
450	406	.392	—		.594	.202	.548	.156	.686	.294
600	542	.522			.823	.301	.823	.301	.868	.346
750	677	.653			1.051	.398	1.051	.398	1.097	.444
900	813	.783			—		1.234	.451	1.280	.497
1050	948	.914					1.371	.457	1.508	.586
1200	1084	1.044					—		1.737	.693
1380									—	

SPECIMEN # 13

$D_i = 0''.171$

$D_o = 0''.251$

$\ell = 0''.5739$

$F = 0.457$

$\psi_C = 1.5258 \times 10^{-3} \ell \, Q_A$

$= 0.8757 \times 10^{-3} Q_A$

$\psi_O = F \times \psi_S$

$= 0.457 \, \psi_S$

$S_T = 1.121 \times 10^{-2} \dfrac{D_o}{D_o^4 - D_i^4} Q_A$

$S_T = 0.903 \, Q_A$

$S_N = \dfrac{F_N}{\pi/4(D_o^2 - D_i^2)}$

Surface finish: 1.0

			RUN # 1		RUN # 2		RUN # 3	
			F_N = 150		F_N = 202		F_N = 350	
			S_N = 10,653		S_N = 14,225		S_N = 21,830	
Q_A	S_T	ψ_C	ψ_O	ψ_R	ψ_O	ψ_R	ψ_O	ψ_R
150	135	.131	.183	.052	.183	.052	.183	.052
300	271	.263	.457	.194	.320	.057	.411	.148
450	406	.394	.594	.200	.594	.200	.548	.154
600	542	.525	—		.640	.115	.777	.252
750	677	.657			.868	.211	.914	.257
900	813	.788			—		1.143	.355
1050	948	.919					1.371	.352
1200							—	

SPECIMEN # 14

$D_i = 0''.171$
$D_o = 0''.251$
$\ell = 0''.5749$
$F = 0.457$

$\psi_C = 1.5258 \times 10^{-3} \ell \, Q_A$

$= 0.8772 \times 10^{-3} \, Q_A$

$\psi_0 = F \times \psi_S$

$= 0.457 \, \psi_S$

$S_T = 1.121 \times 10^{-2} \dfrac{D_o}{D_o^4 - D_i^4} Q_A$

$S_T = 0.903 \, Q_A$

$S_N = \dfrac{F_N}{\pi/4 (D_o^2 - D_i^2)}$

Surface finish: 4/0

			RUN # 1		RUN # 2		RUN # 3	
			F_N = 150		F_N = 202		F_N = 350	
			S_N = 10,653		S_N = 14,325		S_N = 21,830	
Q_A	S_T	ψ_C	ψ_0	ψ_R	ψ_0	ψ_R	ψ_0	ψ_R
150	135	.132	.229	.097	.274	.142	.229	.097
300	271	.263	.548	.285	.457	.194	.411	.148
450	406	.395	.823	.428	.731	.326	.731	.336
600	542	.526	—		1.005	.479	.914	.388
750	677	.658			1.280	.622	1.097	.439
900	813	.789			—		1.371	.582
1050	948	.921					1.600	.679
1200							—	

APPENDIX G

Supplementary Discussion

SUPPLEMENTARY DISCUSSION

In the Discussion of Results it was suggested that the residual deflection could be accounted for by assuming the asperities to act as cantilevers. The following calculations support this theory.

First, neglecting the cantilever effect and assuming the asperities behave as bulk metal leads to a calculated asperity height that is much too large. For example, consider a specimen of the type used in the first part of the work.

Equation (8) in Bowden and Tabor (4) gives

$$A_r = \frac{F_N}{p_m} \qquad (1)$$

A_r = real area of contact

F_N = load normal to surface

p_m = mean pressure over area of contact.

Moreover, the tips of the asperities will deform plastically when

$$p_m = cY$$

c = a constant that depends upon size and shape of asperities. For the surface finishes of our specimens, c = 3.

Y = elastic limit of deformed metal at tip of asperities. Here we will assume the elastic limit equals the proportional limit. Therefore, Y = 60,000 psi for our specimens.

Substituting $p_m = 3(60,000) = 180,000$ psi in equation (1)

$$A_r = \frac{F_N}{180,000} \qquad (3)$$

From strength of materials, we have

$$A = \frac{F_N}{S_N} \qquad (4)$$

A = apparent area of contact (i.e., area of bulk metal at surface)

S_N = normal stress.

From equations (3) and (4)

$$\frac{A_r}{A} = \frac{S_N}{180,000} \qquad (5)$$

Assume the total contact area, A_r, to be a circle of diameter d. This assumption results in the minimum calculated asperity height[*].

---

[*] Consider, instead, that the contact area is an annulus with outside diameter, d_o, and inside diameter, d_i, and an area equal to the real area of contact

$$\therefore \quad \frac{\pi}{4}(d_r)^2 = \frac{\pi}{4}(d_o^{\,2} - d_i^{\,2})$$

It is easily demonstrated from the formulae for twist in a shaft that if equal torques are to cause equal angles of twist,

$$\frac{h}{h_A} = \frac{(d_o^{\,2} - d_i^{\,2})}{(d_o^{\,2} + d_i^{\,2})} \qquad \therefore \quad h < h_A$$

where h is the height of a solid cylinder and h_A is the height of the corresponding annulus.

Therefore,

$$\frac{A_r}{A} = \frac{d_r^{\,2}}{d^2} = \frac{S_N}{180,000} \tag{6}$$

d = diameter of the specimen at the surface.

The area will increase from A_r to A in going from the tips of the asperities to the bulk metal at their roots. In the section on Procedure, an equation was derived for the twist in a cone. Applying this gives

$$\frac{\psi_R}{2} = \frac{5.36 \times 10^{-7}}{k'} \left[\left(\frac{1}{d_r}\right)^3 - \left(\frac{1}{d}\right)^3 \right] Q_A \tag{7}$$

ψ_R = the observed residual twist in minutes. It is divided by two since there are two surfaces in contact at an interface and we are considering the effect for one surface.

$k' = \frac{d - d_r}{2h}$ where h is asperity height.

Q_A = the applied torque in gram inches.

Substitute for k' in equation (7)

$$\psi_R = \frac{4h\,(5.36 \times 10^{-7})}{d - d_r} \left[\left(\frac{1}{d_r}\right)^3 - \left(\frac{1}{d}\right)^3 \right] Q_A \tag{8}$$

Solving equation (8) for h

$$h = \frac{\psi_R\, d^4}{21.44 \times 10^{-7}} \left[\frac{1 - \frac{d_r}{d}}{\left(\frac{d}{d_r}\right)^3 - 1} \right] \frac{1}{Q_A} \tag{9}$$

From equation (6) substitute $\frac{d_r}{d} = \frac{S_N}{180,000}$

$$h = \frac{\psi_R \, d^4}{21,44 \times 10^{-7}} \left[\frac{1 - \left(\frac{S_N}{180,000}\right)^{\frac{1}{2}}}{\left(\frac{180,000}{S_N}\right)^{3/2} - 1} \right] \frac{1}{Q_A} \qquad (10)$$

Based on test data, the following asperity heights were computed using equation (10), to illustrate the orders of magnitude obtained when considering the asperities to behave as bulk metal.

Specimen 9, Run 2.

d = 0.0949", S_N = 63,700 psi, Q_A = 630 gram inches
ψ_R = 0.948 minutes, surface finish: 4/0
$h = 6.12 \times 10^{-3}$ inches.

Specimen 1.

d = 0.125", S_N = 24,500 psi, Q_A = 625.86, ψ_R = 1.229 minutes, surfrace finish: 4/0
$h = 7.50 \times 10^{-3}$ inches

Specimen 8, Run 1.

d = 0.0958, S_N = 69,000 psi, Q_A = 330 gram inches
ψ_R = 0.511 minutes, surface finish: 1.0
$h = 7.20 \times 10^{-3}$ inches.

For purposes of comparison asperity heights were measured with a profilometer. For a 4/0 finish, before applying any normal

load,

$$h = 6.0 \times 10^{-5} \text{ inches}$$

When a normal stress of 21,830 psi was applied

$$h = 4.5 \times 10^{-5} \text{ inches.}$$

When the asperities are considered to act as cantilevers, the calculated height is found to be much closer to the measured. For the purposes of simplification consider an annular specimen such as was used in the second part of investigation.

Starting with equation (3) above,

$$A_r = \frac{F_N}{180,000}$$

The number of asperities in contact can be found by

$$N = \frac{A_r}{A_A} \qquad (11)$$

N = number of asperities in contact

A_A = cross section area of average asperity.

Consider the average asperity to be at the mean diameter of the annulus. The torque action on this asperity is $\frac{Q_A}{N}$ and the force deflecting the asperity is

$$\frac{Q_A}{N\left(\frac{d_m}{2}\right)} \qquad (12)$$

d_m = mean diameter of the annulus.

The maximum deflection of a cantilever due to a load at its free end is

$$\delta = \frac{P\ell^3}{3EI} \qquad (13)$$

δ = maximum deflection of cantilever in inches.

p = load at free end in pounds

I = Moment of inertia of cross section about neutral axis.

ℓ = length of cantilever in inches

E = modulus of elasticity in pounds per square inches.

Rearranging equation (13)

$$\ell^3 = \frac{3EI}{P}\delta \qquad (14)$$

In this case the cantilever length, ℓ, is equal to the asperity height, h, in inches. The deflection, δ, is also equal to one-half the residual twist, ψ_R, (in radians) times the mean radius, $\frac{d_m}{2}$. One-half the residual twist was taken since there are two surfaces in contact that contributed to this displacement of

$$\delta = \frac{\psi_R \text{ (radians)}}{2} \times \frac{d_m}{2} \text{ (inches)} \qquad (15)$$

Assuming that the asperities are of square cross section with an area of 25×10^{-8} cm^2 and substituting equations (3), (11), (12), and (15) into (14), gives

$$\ell^3 = h^3 = 2.67 \times 10^{-8} \frac{\psi_R F_N}{\sqrt{A}} (d_m)^2 \text{ (inches)}^3 \qquad (16)$$

Where, d_m = 0.211 inches for the specimens tested.

Taking data for Specimen # 14, Run 3, as an example,

F_N = 310# ψ_R = 0.582 minutes

Q_a = 900 qm-inches, surface finish = 4/0

Substituting the above data into equation (16) gives the following estimate of asperity height.

$h = 6.02 \times 10^{-4}$ inches.

In contrast, the asperity height as measured by profilometer and as calculated by equation (10) are 4.5×10^{-5} and 7.5×10^{-3} inches respectively.

As an additional point of interest, the maximum bending stress can be calculated using the data for specimen 14, Run 3, above.

$$\sigma_B = \frac{M_c}{I} = \frac{phc}{I} \qquad (17)$$

σ_B = maximum bending stress in a cantilever due to load p.

c = maximum distance from neutral axis = 2.5×10^{-4} cm for the assumed asperity.

Other quantities, as defined in equation (13).

Substituting equations (3), (11), and (12) into equation (17) and solving,

$\sigma_B = 1.97 \times 10^{5}$ psi.

This is further support to the theory that asperities can withstand much larger stress than the bulk metal and still retain elastic properties.

APPENDIX B

Original Data

	SPECIMEN # 1 Surface Finish: 4/0	SPECIMEN # 2 Surface Finish: 2/0	SPECIMEN # 3 Surface Finish: 4/0		SPECIMEN # 4 Surface Finish: 4/0
	R U N # 1 S_N = 24,500	R U N # 1 S_N = 53,750	R U N # 1 S_N = 44,600	R U N #2 S_N = 52,100	R U N # 1 S_N = 58,200
S_A	ψ_S	ψ_S	ψ_S	ψ_S	ψ_S
145.86	1.0	1.6	2.0	2.1	1.0
205.86	1.3	——	3.0	3.0	1.4
217.86	——	2.2	——	——	——
265.86	1.8	——	4.0	3.9	1.9
325.86	2.1	——	5.0	——	——
337.86	——	4.2	——	4.9	2.2
385.86	2.5	——	—	5.9	2.8
396.34	——	5.0		——	——
445.86	2.9	——		—	3.1
458.82	——	5.8			——
505.86	3.2	——			—
519.30	——	—			
565.86	3.8				
625.86	4.1				
685.86	4.5				
745.86	—				

SPECIMEN # 5

Surface Finish: Ø

	RUN # 1	RUN # 2	RUN # 3	RUN # 4	RUN # 5	RUN # 6	RUN # 7
	S_N = 67,800	S_N = 73,300	S_N = 78,100	S_N = 83,800	S_N = 93,400	S_N = 105,000	S_N = 115,200
S_A	ψ_S	ψ_S	ψ_S	ψ_S	ψ_S	ψ_S	ψ_S
150	1.6	1.6	1.6	1.6	1.6	1.6	1.2
210	2.1	2.1	2.1	2.1	2.1	2.1	1.9
270	3.0	3.0	3.0	3.0	2.8	2.8	2.5
330	3.9	3.9	3.7	3.5	3.3	3.2	3.0
390	4.7	4.4	4.1	4.0	3.9	4.0	3.7
450	—	5.0	4.9	4.8	4.7	4.5	4.1
510		5.8	5.6	5.4	5.3	5.0	—
570		6.5	6.3	6.0	6.0	—	
630		—	—	—	—		

SPECIMEN # 6

Surface Finish: As Machined

	RUN # 1	RUN # 2	RUN # 3	RUN # 4	RUN # 5
	S_N = 62,500	S_N = 67,450	S_N = 72,600	S_N = 77,800	S_N = 83,000
σ_a	ψ_s	ψ_s	ψ_s	ψ_s	ψ_s
150	1.6	1.6	1.6	1.7	1.6
210	2.2	2.2	2.2	2.3	2.3
270	3.0	3.0	3.0	2.9	2.9
330	4.0	3.7	3.7	3.7	3.6
390	—	4.4	4.3	4.4	4.3
450		—	5.0	5.0	4.9
510			—	5.8	5.6
570				—	6.3
630					—

SPECIMEN # 6 (continued)

Surface Finish: As Machined

	RUN # 6 S_N = 88,200	**RUN # 7** S_N = 91,300	**RUN # 8** S_N = 94,150	**RUN # 9** S_N = 95,500	**RUN # 10** S_N = 108,000
q_1	ψ_s	ψ_s	ψ_s	ψ_s	ψ_s
150	1.4	1.3	1.6	1.6	1.5
210	2.3	2.2	2.3	2.1	2.8
270	—	2.8	2.9	2.9	4.0
330		3.3	—	3.6	4.7
390		—		4.1	—
450				4.8	
510				—	
570					
630					

	SPECIMEN # 7 Surface Finish: # 3			SPECIMEN # 8 Surface Finish: 1.0	
	RUN # 1 $S_N = 56{,}000$	RUN # 2 $S_N = 63{,}700$	RUN # 3 $S_N = 70{,}800$	RUN # 1 $S_N = 48{,}400$	RUN # 2 $S_N = 55{,}400$
S_A	ψ_S	ψ_S	ψ_S	ψ_S	ψ_S
150	2.8	2.8	2.8	2.2	2.2
180	——	3.2	3.3	——	——
210	3.9	3.8	3.8	3.2	3.3
240	——	4.2	4.3	——	——
270	5.0	4.9	4.9	4.3	4.3
300	——	5.4	—	——	——
330	6.1	5.9		5.4	5.3
360	——	6.5		——	——
390	—	7.1		—	6.3
420		—			——
450					7.3
510					8.3
570					—

SPECIMEN # 8

Surface Finish: 1.0

	R U N # 3	R U N # 4	R U N # 5	R U N # 6	R U N # 7
	$S_N = 69,000$	$S_N = 76,100$	$S_N = 83,100$	$S_N = 110,900$	$S_N = 124,800$
C_A	ψ_S	ψ_S	ψ_S	ψ_S	ψ_S
150	2.2	2.2	2.2	2.2	2.2
180	——	——	——	——	——
210	3.2	3.1	3.1	3.1	3.0
240	——	——	——	——	——
270	4.1	4.0	4.1	3.9	3.8
300	——	——	——	——	——
330	5.1	5.1	5.0	4.8	—
360	——	——	——	——	
390	6.1	5.9	5.9	—	
420	——	——	——		
450	7.0	6.9	—		
510	8.0	7.9			
570	—	—			

SPECIMEN # 9

Surface Finish: 4/C

	RUN # 1	RUN # 2	RUN # 3
	S_N = 49,500	S_N = 63,700	S_N = 70,800
S_A	ψ_S	ψ_S	ψ_S
150	2.2	2.1	2.0
210	3.0	3.1	—
270	4.2	4.0	4.0
330	5.4	5.0	—
390	6.6	6.0	6.0
450	—	7.0	—
510		8.0	8.0
570		9.0	—
630		10.0	10.0
690		11.0	—
750		12.0	11.9
810		13.0	—
870		—	
930			
990			
1050			
1110			
1170			
1290			
1470			

SPECIMEN # 1C

Surface Finish: 1.0

	RUN # 1	**RUN # 2**	**RUN # 3**	**RUN # 4**	**RUN # 5**	**RUN # 6**
	S_N = 24,500	S_N = 32,600	S_N = 40,800	S_N = 48,900	S_N = 57,030	S_N = 73,400
S_A	ψ_S	ψ_S	ψ_S	ψ_S	ψ_S	ψ_S
150	2.0	2.0	1.9	1.8	1.8	1.7
210	2.8	2.7	—	2.4	—	2.3
270	3.6	3.4	3.2	3.1	3.1	3.0
330	4.4	4.1	—	4.0	—	3.9
390	—	4.8	4.6	4.6	4.6	4.5
450		5.6	—	5.4	—	—
510		6.2	6.1	—	6.0	5.9
570		6.9	—	6.9	—	—
630		7.7	7.6	—	7.5	7.2
690		8.4	—	8.2	—	—
750		9.2	9.2	—	8.9	8.7
810		10.0	—	9.6	—	—
870		—	—	—	10.3	10.0
930				11.2	—	—
990				—	11.7	11.4
1050				12.4	—	—
1110				—	—	—
1170				14.0		
1290				15.3		
1470				—		

	SPECIMEN # 11 Control Specimen for Runs on Annuli		SPECIMEN # 12 Surface Finish: As Machined			
	RUN # 1	RUN # 2	RUN # 1	RUN # 2	RUN # 3	RUN # 4
	S_N = 10,653	S_N = 21,830	S_N = 10,653	S_N = 14,225	S_N = 17,605	S_N = 21,830
S_A	ψ_s	ψ_s	ψ_s	ψ_s	ψ_s	ψ_s
150	0.3	0.2	0.5	0.4	0.3	0.5
300	0.5	0.6	1.0	0.9	0.7	1.0
450	0.8	0.8	—	1.3	1.2	1.5
600	1.1	1.2		1.8	1.8	1.9
750	—	1.5		2.2	2.2	2.4
900		1.6		—	2.7	2.8
1050		2.0			3.0	3.3
1200		—			—	3.8
1380						4.0
1500						4.5
1620						—

	SPECIMEN # 13 Surface Finish: 1.0			SPECIMEN # 14 Surface Finish: 4/0		
	RUN # 1 S_N = 10,653	RUN # 2 S_N = 14,225	RUN # 3 S_N = 21,830	RUN # 1 S_N = 10,653	RUN # 2 S_N = 14,225	RUN # 3 S_N = 21,830
q_s	ψ_s	ψ_s	ψ_s	ψ_s	ψ_s	ψ_s
150	0.4	0.4	0.4	0.5	0.6	0.5
300	1.0	0.7	0.9	1.2	1.0	0.9
450	1.3	1.3	1.2	1.8	1.6	1.6
600	—	1.4	1.7	—	2.2	2.0
750		1.9	2.0		2.8	2.4
900		—	2.5		—	3.0
1050			3.0			3.5
1200			—			—

DEFINITION OF SYMBOLS

A	Area (square inches).
d	Diameter of solid specimens at interface (inches).
D	Maximum diameter of solid specimens (inches).
D_i	Inner diameter of annular specimens (inches).
D_o	Outer diameter of annular specimens (inches).
E	Young's modulus of elasticity (pounds per square inch).
F	Conversion factor to convert angle of twist in scale units, ψ_s , to minutes, ψ_o .
F_N	Force normal to interface (pounds).
G	Modulus of rigidity (pounds per square inch).
k	$\frac{R - r_o}{\ell}$
ℓ	Length along solid specimens from $r = r_o$ to $r = R$ (inches).
ℓ	Length along annular specimens between indicator arms, (inches).
ℓ'	Length along solid specimens between base of bevel and indicator arm.
Q_A	Applied torque (gram inches).
r	Radius (inches).
r_o	Radius of solid specimens at interface (inches).
R	Maximum radius of solid specimens (inches).
S_N	Normal stress at interface due to F_N. S_N equals F_N/A divided by the area of the interface, (psi).
S_T	Maximum tangential stress in interface due to applied torque, (psi).

T_A	Torque applied to specimen (pound inches).
T_E	Effective torque at interface (pound inches) = $\frac{T_A}{2}$
σ	Shear stress (psi).
$\frac{d\phi}{dx}$	Angle of twist (radians) per unit length (inches).
ϕ_{ℓ}	Total theoretical angle of twist in length ℓ. (radians).
$\phi_{\ell'}$	Total theoretical angle of twist in length ℓ'. (radians).
ψ_c	Calculated corresponding angle of twist in minutes for annular specimens if there had been no interface between arms.
ψ_{ℓ}	Total theoretical angle of twist in length ℓ. (minutes).
$\psi_{\ell'}$	Total theoretical angle of twist in length ℓ'. (minutes).
ψ_o	Observed angle of twist (minutes).
ψ_R	Residual angle of twist or residual twist (minutes).
ψ_s	Observed angle of twist in scale units of the optic vernier.
ψ_T	Total theoretical angle of twist in length $\ell + \ell'$. (minutes).

BIBLIOGRAPHY

1. G. A. Tomlinson, P. L. Thorpe, and J. H. Gough - "An Investigation of the Fretting Corrosion of Closely Fitting Surfaces" - I. Mech. E. 1939.

2. F. P. Bowden and D. Tabor - "The Friction and Lubrication of Solids" - Oxford at the Clarendon Press, 1950.

3. C. Herring and J. K. Galt - "Elastic and Plastic Properties of Very Small Metal Specimens" - The Physical Review, Volume 85, p. 1060-1061, March 15, 1952.

4. S. Timoshenko and G. H. MacCullough - "Elements of Strength of Materials" - D. Van Nostrand Company, Inc., New York, 1940.

5. M. C. Shaw and E. F. Macks - "Analysis and Lubrication of Bearings", McGraw-Hill Book Company, Inc., New York, 1949.

6. A. H. Cottrell - "Dislocations and Plastic Flow in Crystals", Oxford at the Clarendon Press, 1953.

7. L. P. Tarasov - "Relation of Surface-Roughness Readings to Actual Surface Profile" - ASME, Transactions, p. 189, 1945.

Thesis
C77 28805
Coyle, Arthur J.
Elastic behavior of
metal interfaces.

Thesis
C77 28805
Coyle, Arthur J.
Elastic behavior of metal
interfaces.

thesC77
Elastic behavior of metal interfaces /
3 2768 002 09034 2
DUDLEY KNOX LIBRARY